3D Perspective Conversion in Excel – Part #1 – by George Lungu

In computer graphics we often need to be able to display a three-dimensional image in two dimensions and preserve the perspective appearance. If we walk on a straight road, it appears that the road narrows with the distance. This is the perspective effect and it is a result of mapping a three-dimensional image on a two-dimensional surface (i.e. a computer monitor, a screen, or the retina of the eye).

This tutorial is not about stereoscopic view, which refers to a technique for enhancing the illusion of depth in an image by presenting two offset images separately to the left and right eye of the viewer.

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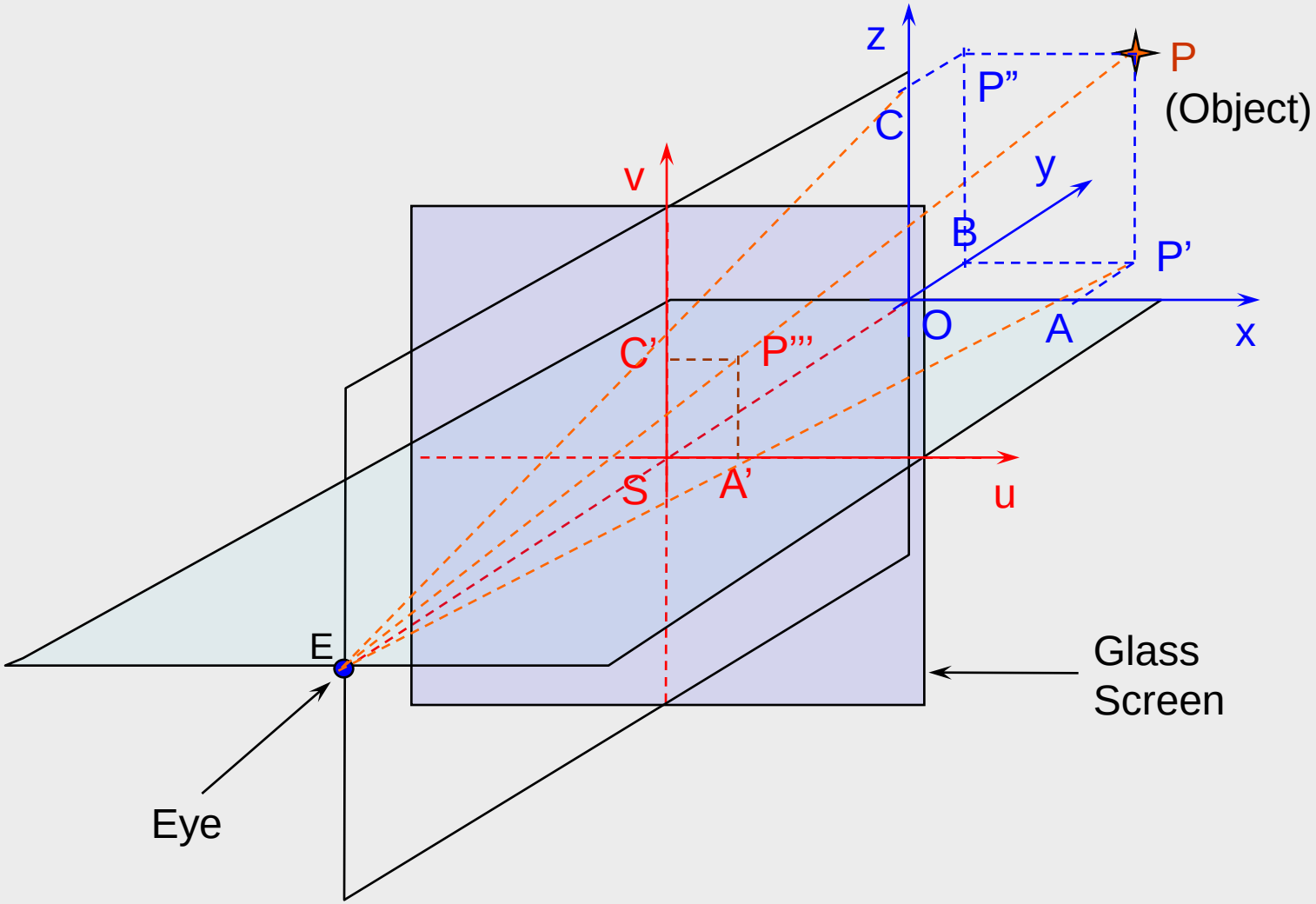
The setup:

- Let’s assume we are watching point “P” of a 3D scene through a piece of glass screen



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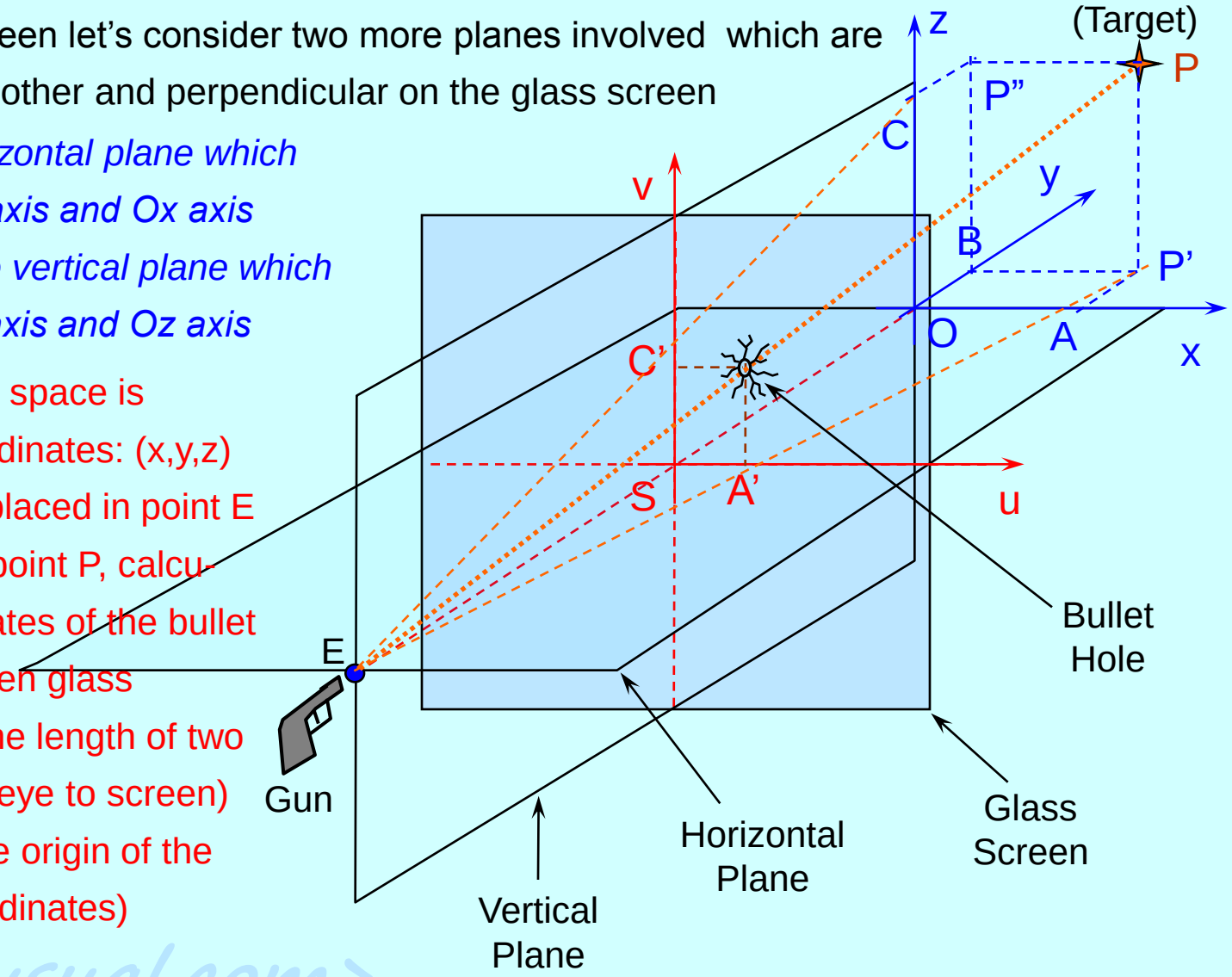
- We define the “object system of coordinate” which is a 3D system of coordinates (x,y,z) in which point P (object) is situated
- We also define a 2D system of coordinates (u,v) on the glass screen (screen system)
- Besides the glass screen let’s consider two more planes involved which are perpendicular on each other and perpendicular on the glass screen

- There is the horizontal plane which contains the O’u axis and Ox axis

- There is also the vertical plane which contains the O’v axis and Oz axis

- Point P in the object space is defined by three coordinates: (x,y,z)
- **Goal:** Using a gun placed in point E to precisely shoot at point P, calculate the (u,v) coordinates of the bullet hole through the screen glass knowing (x,y,z) and the length of two more segments: ES (eye to screen) and SO (screen to the origin of the object system of coordinates)



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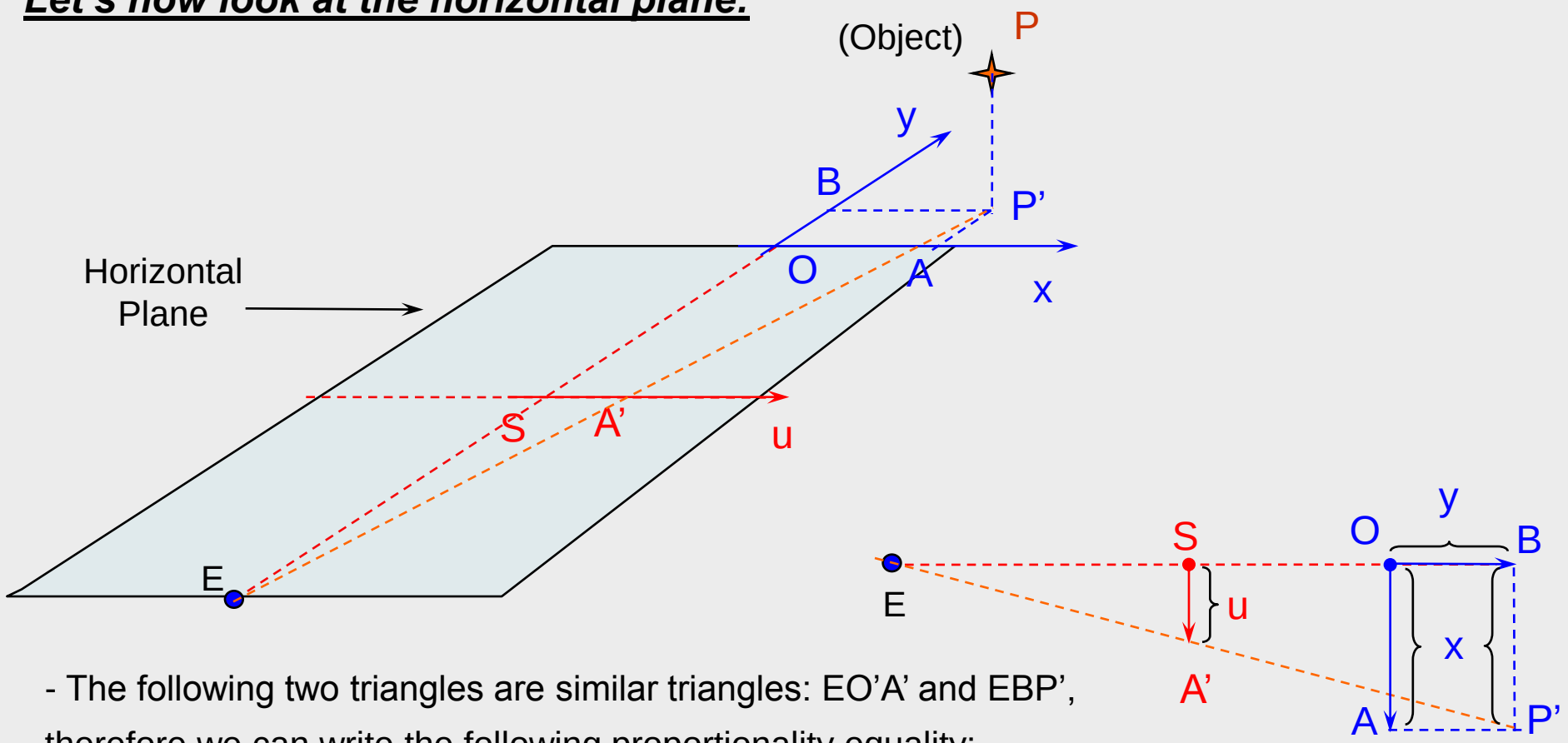
Let's now look at the horizontal plane:



- The following two triangles are similar triangles: EO'A' and EBP', therefore we can write the following proportionality equality:

$$\frac{ES}{EB} = \frac{SA'}{BP'}$$

And from here we can calculate "u":

$$u = \frac{x \cdot ES}{ES + SO + y}$$

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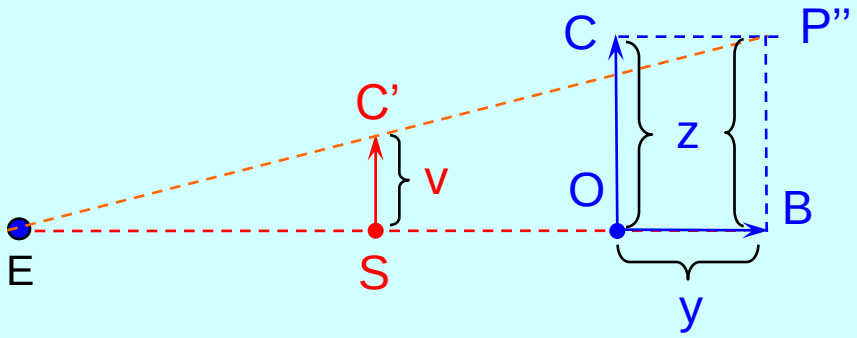
We can do the same type of analysis in the vertical plane:



- The following two triangles are similar triangles: ESC' and EBP'', therefore we can write the following proportionality equality:

$$\frac{ES}{EB} = \frac{SC'}{BP''}$$

And from here we can calculate "v":

$$v = \frac{z \cdot ES}{ES + SO + y}$$

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In conclusion we have the following conversion equations:

$$u = \frac{x \cdot ES}{ES + SO + y}$$

$$v = \frac{z \cdot ES}{ES + SO + y}$$

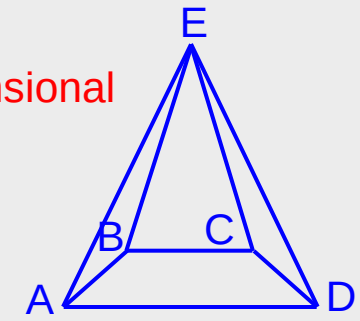
Where:

- ES is the distance between eye and screen
- SO is the distance between the screen and the origin of the object system of coordinates

The object has to be in front of the observer therefore EB (ES+SO+y>0) must be positive at all times to prevent artifacts from being displayed on the screen. While displaying multipoint shapes, the perspective conversion program has to be written as to *eliminate any shape* that has even one vertex which does not satisfy the previous condition.

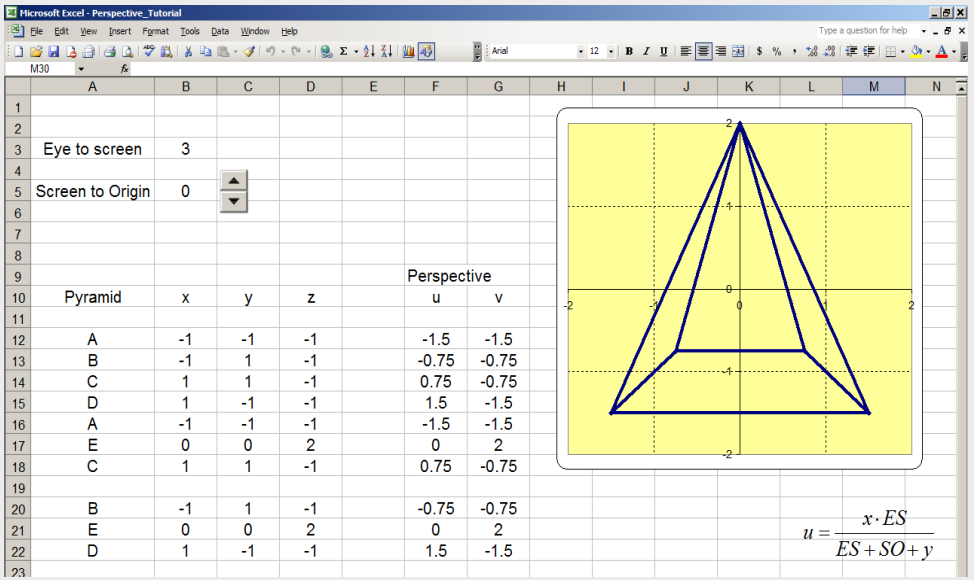
Let's create a sample worksheet to verify the concepts covered here:

- Range (B12:D22) contains the 3-dimensional data for building the following pyramid



- The distance eye-screen is constant, set at 3
- The distance screen-origin can be adjusted within the range [-3, 7] using a spinner button and an associated macro

- Range (F12:G22) contains the 3D to 2D conversion formulas described earlier in this presentation:
- F12: "=B12*B$3/(B$3+B$5+C12)"
- G12: "=D12*B$3/(B$3+B$5+C12)"
- Copy down F12:G12 to F22:G22
- Adjust the screen-origin distance and see how the pyramid changes position
- See what happens when the screen-origin distance gets too small
- Also verify how decreasing the eye-screen distance increases the perspective effect



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