Creating a basic planetary system in Excel

by

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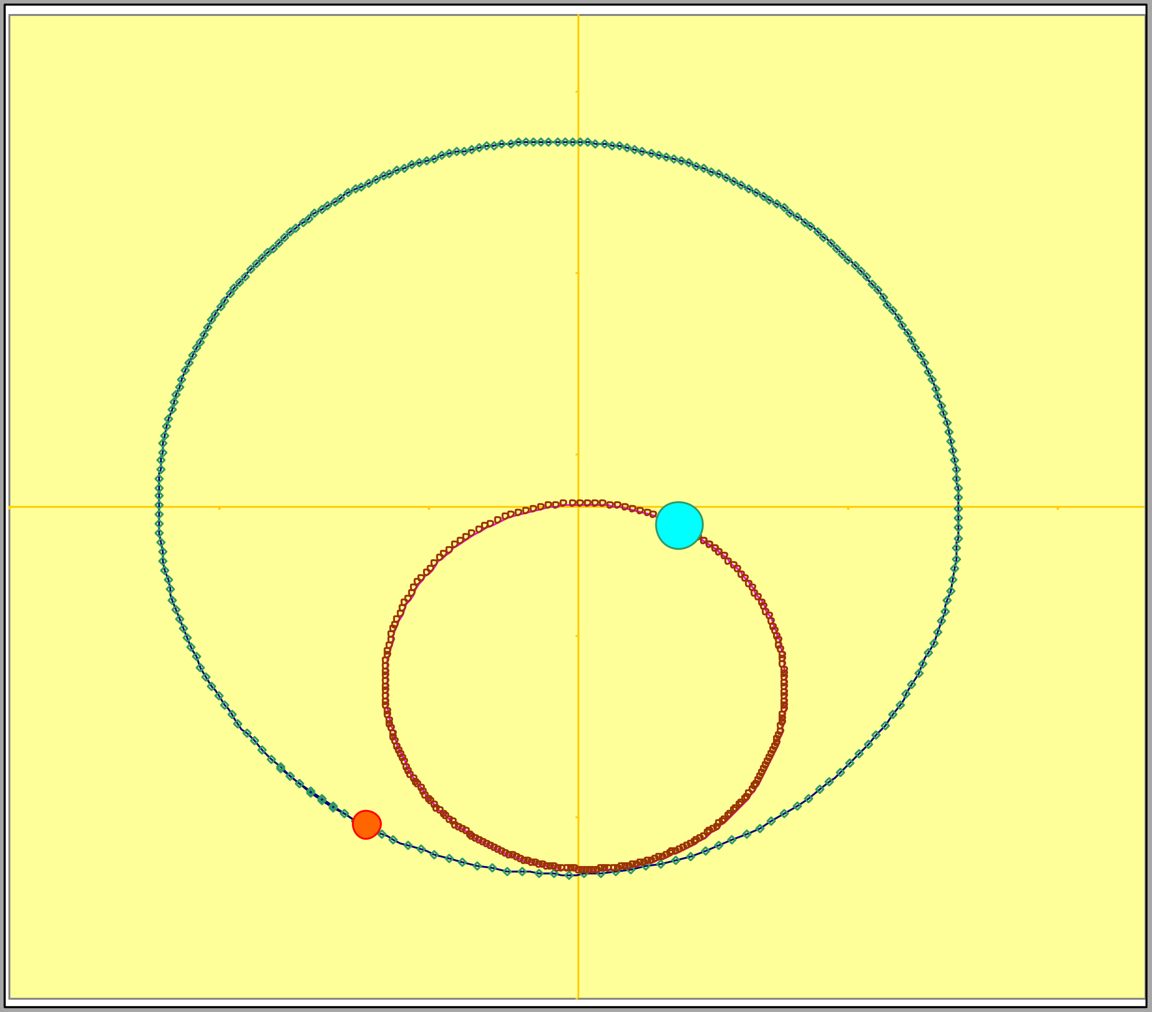
* For the model presented here it is recommended that you use Excel 2003 or older. The 2007 will be either excessively slow.

We want to create the following model:



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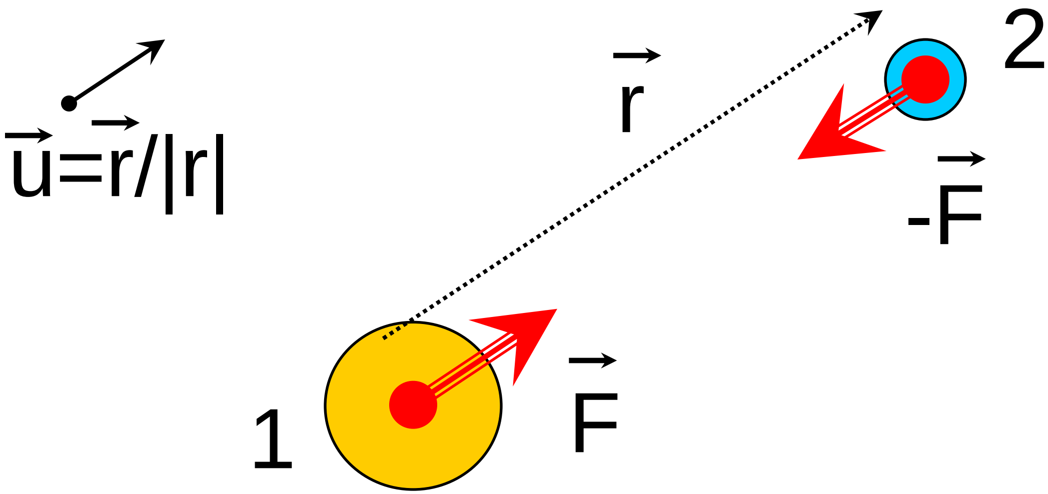
Some basic theory from high school



$\vec{F} = m*\vec{a}$ – Newton's 2nd law

$\vec{F} = \vec{u}$ *k*m1*m2 / r^2 – Newton's universal attraction formula

That was not a very rigorous. Pay attention to few details:

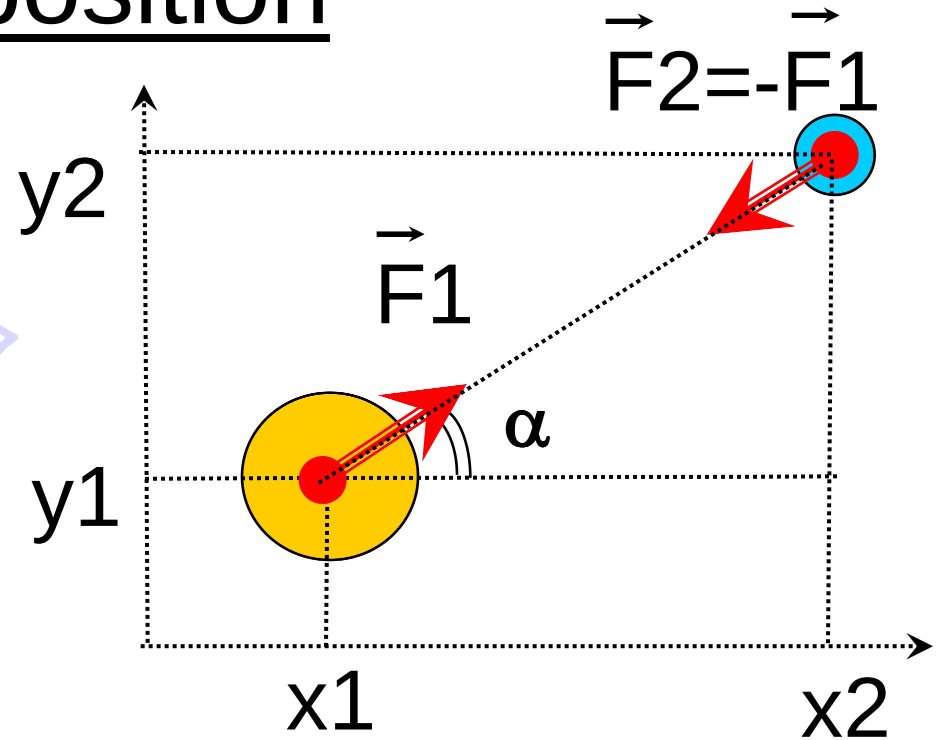
1.- The forces acting on each body have the same direction and absolute value but opposite sense

2.- We are talking attraction not repulsion!

3.- The acceleration has the same direction and sense with the force or gravity.

4. Of course because this problem is in 2-dimens-sional we need to solve it separately along x and y axes

2D vector decomposition



$$F1x = F1 \cdot \cos(\alpha)$$

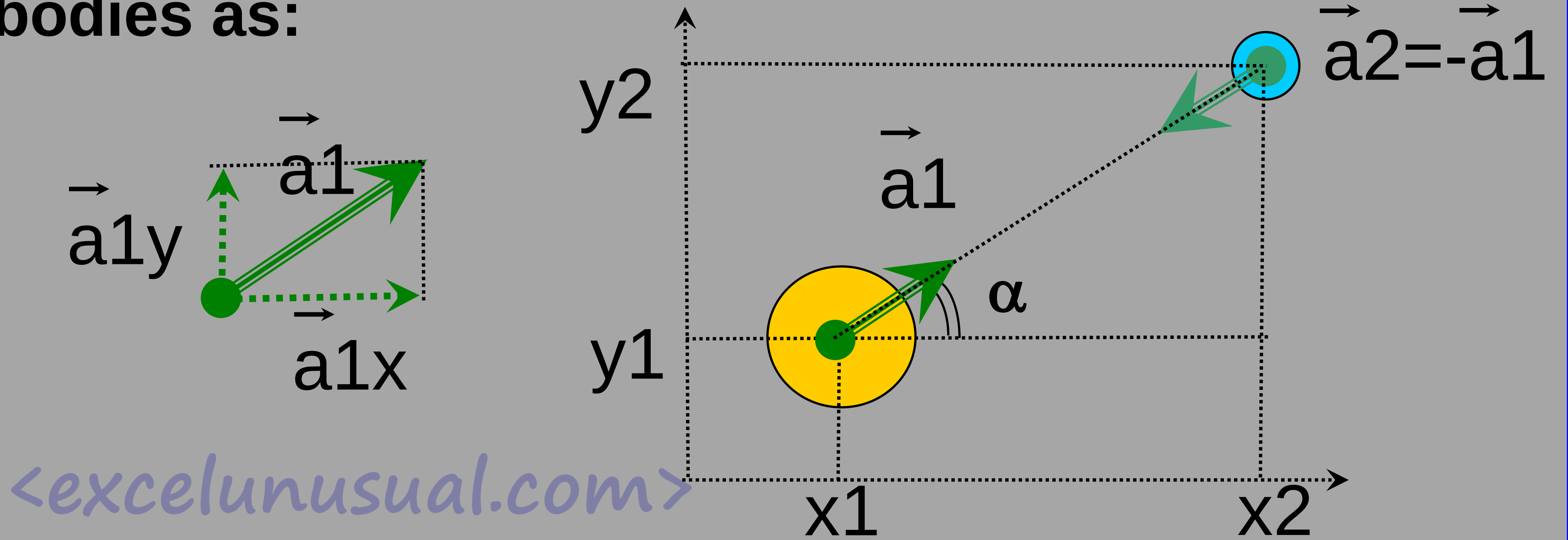
$$F1y = F1 \cdot \sin(\alpha)$$

$$F1 = k \cdot m1 \cdot m2 / ((y2-y1)^2+(x2-x1)^2)$$

$$\cos(\alpha) = (x2-x1) / \mathrm{sqrt}((y2-y1)^2+(x2-x1)^2)$$

$$\sin(\alpha) = (y2-y1) / \mathrm{sqrt}((y2-y1)^2+(x2-x1)^2)$$

From Newton's second law (F = m*a) and the decomposition formulas we can therefore write the x and y components of the accelerations of both bodies as:



a1x = k*m2* (x2-x1) / ((y2-y1)^2+(x2-x1)^2)^(3/2)

a1y = k*m2* (y2-y1) / ((y2-y1)^2+(x2-x1)^2)^(3/2)

a2x = k*m1* (x1-x2) / ((y2-y1)^2+(x2-x1)^2)^(3/2)

a2y = k*m1* (y1-y2) / ((y2-y1)^2+(x2-x1)^2)^(3/2)

We also know the definition of acceleration as the derivative of speed with respect to time:

$a_x = dv_x/dt$ or $a_y = dv_y/dt$

for a very small time increment we can assume that the acceleration is constant and re-write :

$a_{x\_current} = (v_{x\_current} - v_{x\_previous})/dt$

$a_{y\_current} = (v_{y\_current} - v_{y\_previous})/dt$

From here we can derive the following:

$v_{x\_current} = v_{x\_previous} + a_{x\_current} * dt$

$v_{y\_current} = v_{y\_previous} + a_{y\_current} * dt$

<- Very important

Similarily

$v_x = dx/dt$ or $v_y = dy/dt$

for a very small time increment we can assume that the speed is constant and re-write :

$$v_{x\text{-current}} = (x_{current} - x_{previous})/dt$$
$$v_{y\text{-current}} = (y_{current} - y_{previous})/dt$$

From here we can derive the following:

$$x_{current} = x_{previous} + v_{x\text{-current}} * dt$$
$$y_{current} = y_{previous} + v_{y\text{-current}} * dt$$

<- Very important

Outline:

$$\begin{cases} a1x = k*m2*\ (x2-x1)\ /\ ((y2-y1)\verb|^|2+(x2-x1)\verb|^|2)\verb|^|(3/2) \\ a1y = k*m2*\ (y2-y1)\ /\ ((y2-y1)\verb|^|2+(x2-x1)\verb|^|2\ )\verb|^|(3/2) \\ a1x = k*m1*\ (x1-x2)\ /\ ((y2-y1)\verb|^|2+(x2-x1)\verb|^|2\ )\verb|^|(3/2) \\ a1y = k*m1*\ (y1-y2)\ /\ ((y2-y1)\verb|^|2+(x2-x1)\verb|^|2\ )\verb|^|(3/2) \end{cases}$$

$$v_{x\_current} = v_{x\_previous} + a_{x\_current} *dt$$

$$v_{y\_current} = v_{y\_previous} + a_{y\_current} *dt$$

$$x_{current} = x_{previous} + v_{x\text{-}current} *dt$$

$$y_{current} = y_{previous} + v_{y\text{-}current} *dt$$

The above formulas are all we need to build a planetary system

So how is the differential equation system solved?

In the previous page you can see 3 different categories of formulas:

- Acceleration/Force calculations -> yellow formulas (any time: a=F/m or F=ma)

- Speed calculation -> green formulas

- Coordinate calculations -> blue formulas

A pure spread sheet type solution

The time in this case time will increase down the column. Each row will represent a discrete moment of time.

Advantages of the pure spreadsheet solution:

1. Requires practically no VBA (Visual Basic for Applications).

2. It is very fast

There are 2 disadvantage of the tabular solution:

1. The number of time steps is limited

2. The files are large

As we mentioned before, time advances vertically along the column.
The arrows and their colors relate to each type of equation:

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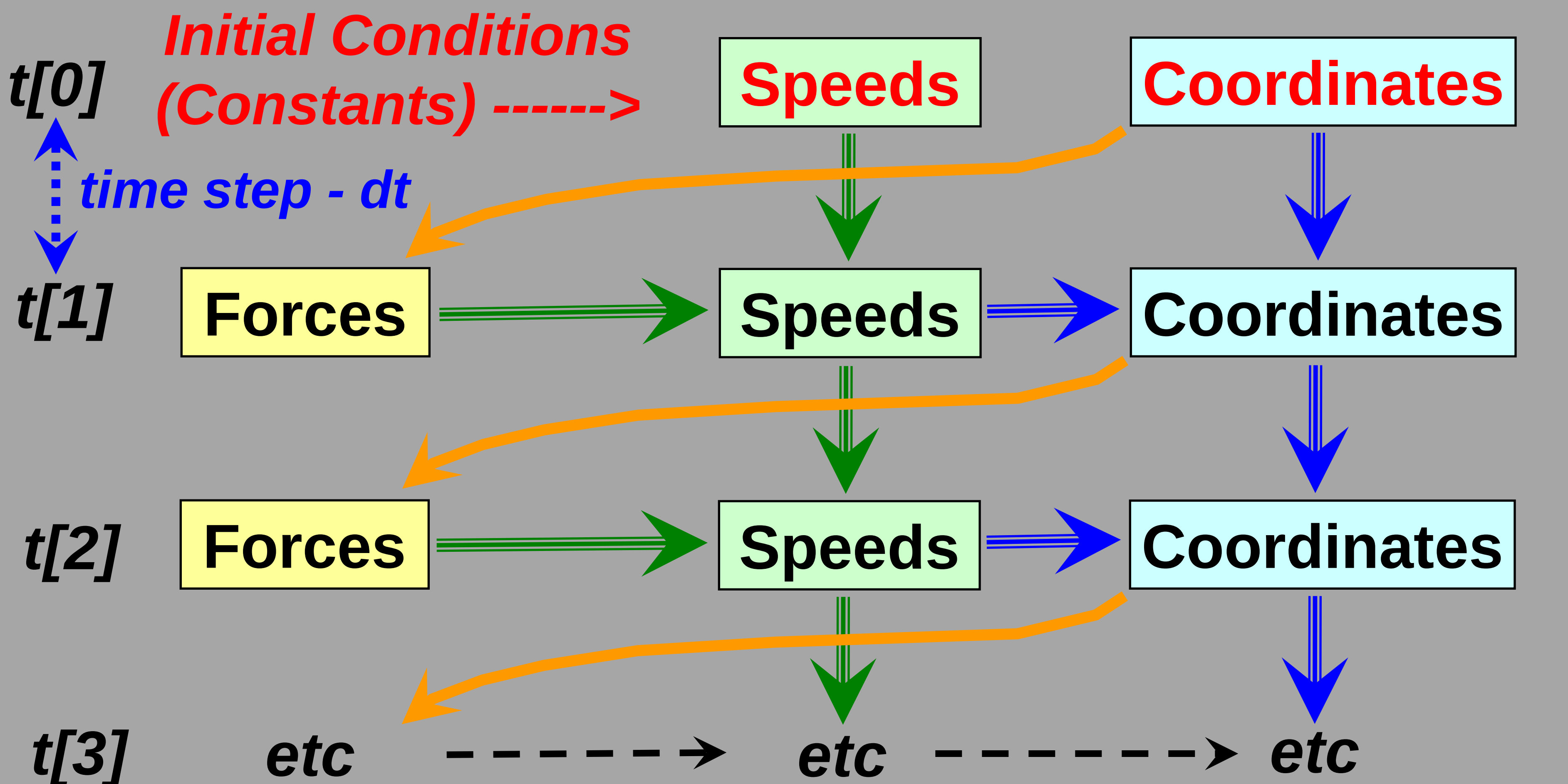

Some important points:

The forces at time *t[n]* are calculated from the coordinates at time *t[n-1]* -> this is a compromise which gives decent solutions as long as *dt* is small enough (see the yellow formulas)

The speeds are calculated from the previous speeds (at *t[n-1]*) and the current accelerations (see the green formulas)

The coordinates are calculated from the previous coordinates and the current speeds (see the blue formulas)

A sequential type solution

There is just one row of calculations. A basic VBA macro will create an infinite loop.

Advantages of the pure spreadsheet solution:

1. It can run forever. It can be stopped and restarted.

2. The Excel files are small.

The only disadvantage of the sequential solution is that it is slower than the previous (instantaneous) version. In our case this does not matter since the simulation will be display limited (around 20- 40 frames per second depending on the computer used and the chart size).

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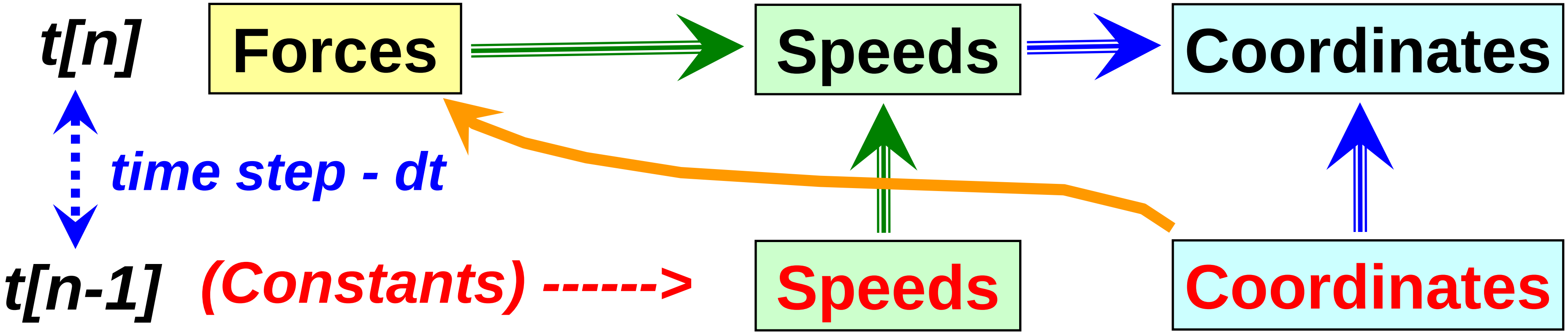
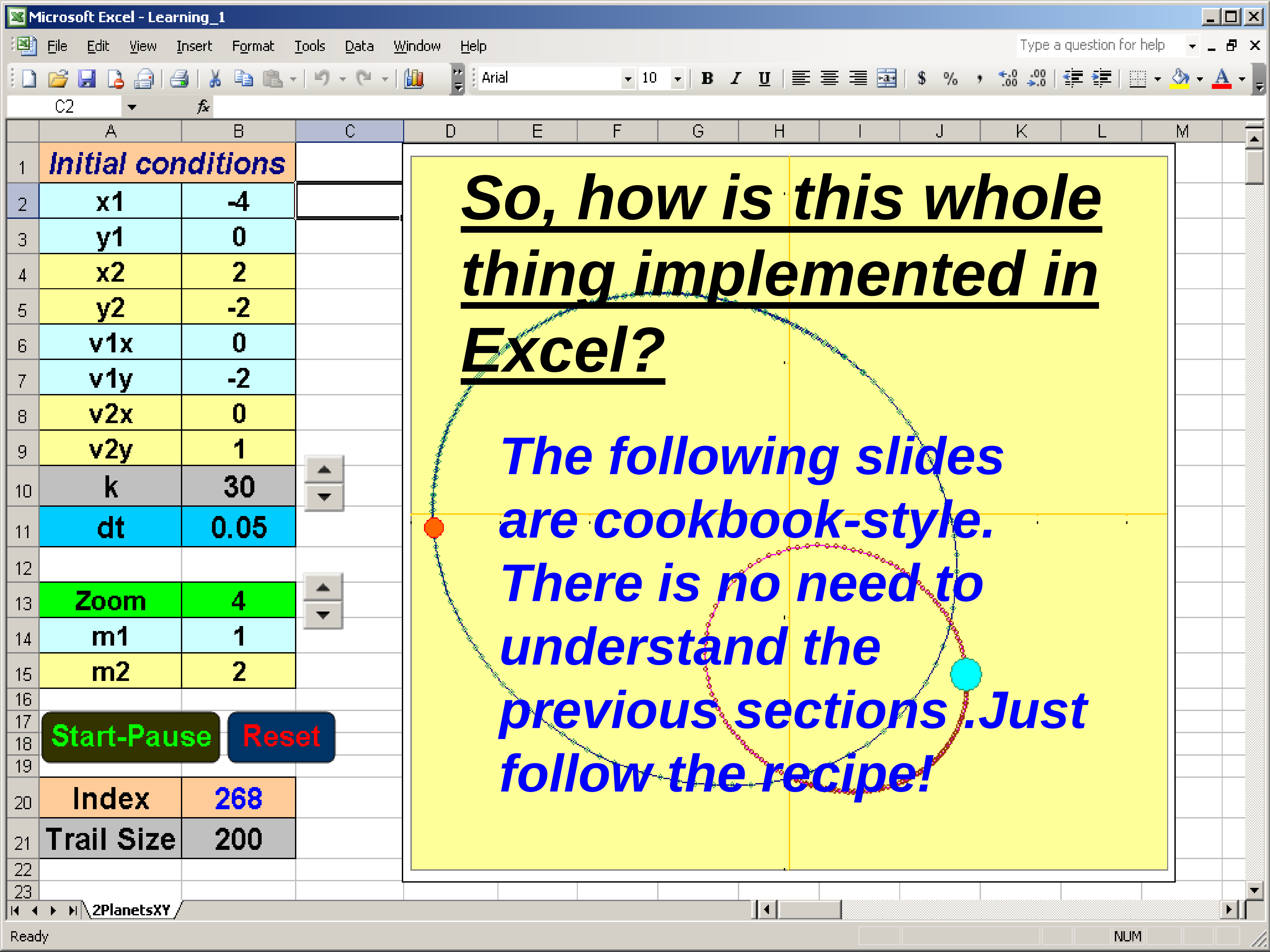
The macro will copy the speed and coordinate values from the active row (*t[n]*) to the following row.

The arrows and their colors relate to each type of equation:



The speeds and coordinates at *t[n-1] are copied from the* speeds and coordinates at *t[n]. Therefore the formulas are using their old results to calculate the results in an infinite loop.*



Microsoft Excel - Learning_1
File Edit View Insert Format Tools Data Window Help
Type a question for help
Arial
10
C2
Initial conditions
x1 -4
y1 0
x2 2
y2 -2
v1x 0
v1y -2
v2x 0
v2y 1
k 30
dt 0.05
Zoom 4
m1 1
m2 2
Start-Pause
Reset
Index 268
Trail Size 200
So, how is this whole thing implemented in Excel?
The following slides are cookbook-style. There is no need to understand the previous sections .Just follow the recipe!
2PlanetsXY
Ready
NUM

	A	B
1	*Initial conditions*	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	30
11	dt	0.05
12		
13	Zoom	5
14	m1	1
15	m2	2
16		
17		
18		
19		
20	Index	0
21	Trail Size	200

Create the input data area

Cell "A1": Initial Conditions

Cell "A2": x1 Cell "A3": y1
Cell "A4": x2 Cell "A5": y2
Cell "A6": v1x Cell "A7": v1y
Cell "A8": v2x Cell "A9": v2y

Cell "A10": k
Cell "A11": dt
Cell "A13": Zoom
Cell "A14": m1
Cell "A15": m2
Cell "A20": Index
Cell "A21": Trail Size

	A	B
1	*Initial conditions*	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	30
11	dt	0.05
12		
13	Zoom	5
14	m1	1
15	m2	2
16		
17		
18		
19		
20	Index	0
21	Trail Size	200

Input data area - figures

I use font size 16 / bold

The only formula is in the cell “B20”: = L27/B11-1

For the rest of the cells just plug in those numbers as a starter.

	A	B
1	*Initial conditions*	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	30
11	dt	0.05
12		
13	Zoom	5
14	m1	1
15	m2	2
16		
17		
18		
19		
20	Index	0
21	Trail Size	200

Create spin button "k"

This button will change the gravitational constant on-the-fly

View -> Toolbars -> Control Toolbox -> Spin Button (drag create)

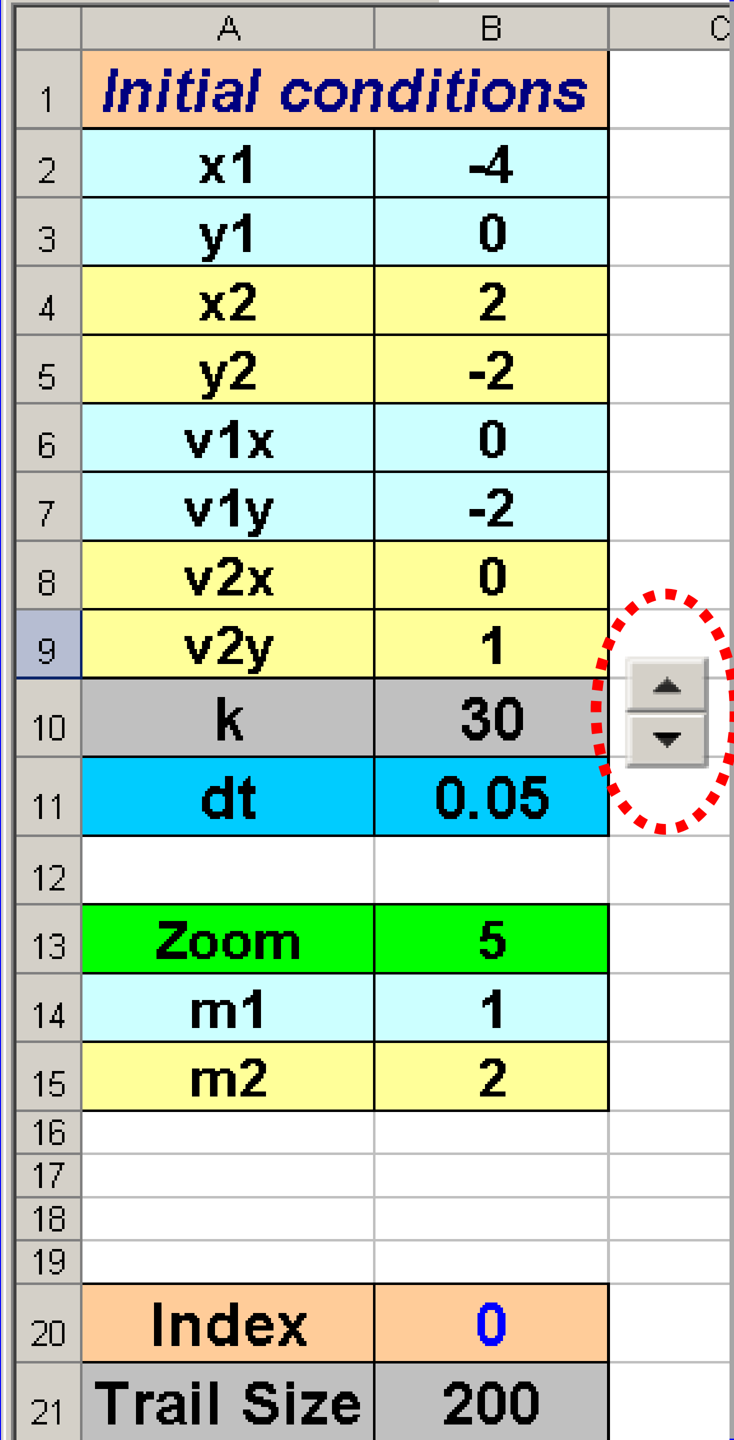
In the control toolbox hit "Design Mode" then right click the button and select "Properties"

Change the following:

(Name): k

Min: 0

Max: 200



	A	B
1	Initial conditions	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	30
11	dt	0.05
12		
13	Zoom	5
14	m1	1
15	m2	2
16		
17		
18		
19		
20	Index	0
21	Trail Size	200

Macro associated to button "k"

```
Sub k_Change()
Range("B10") = k.Value
End Sub
```

	A	B	C
1	*Initial conditions*		
2	x1	-4	
3	y1	0	
4	x2	2	
5	y2	-2	
6	v1x	0	
7	v1y	-2	
8	v2x	0	
9	v2y	1	
10	k	30	
11	dt	0.05	
12			
13	Zoom	4	
14	m1	1	
15	m2	2	
16			
17			
18			
19			
20	Index	268	
21	Trail Size	200	

Create spin button "Zoom"

This button will change the Zoom on-the-fly

View -> Toolbars -> Control Toolbox -> Spin Button (drag create)

Right click the button and select "Properties"

Change the following:

(Name): Zoom

Min: 1

Max: 50

Macro associated to button "Zoom"

```
Private Sub Zoom_Change()
Range("B13") = Zoom.Value ^ (1 + Zoom.Value / 100)
With ActiveSheet.ChartObjects("Chart 1").Chart
    .Axes(xlCategory).MinimumScale = - Range("B13")
    .Axes(xlCategory).MaximumScale = Range("B13")
    .Axes(xlValue).MinimumScale = - Range("B13")
    .Axes(xlValue).MaximumScale = Range("B13")
End With
Range("C17").Select
End Sub
```

	A	B
1	*Initial conditions*	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	40
11	dt	0.05
12		
13	Zoom	4
14	m1	1
15	m2	2
16		
17		
18		
19		
20	Index	853
21	Trail Size	60

Create spin button “Trail Size”

This button will change the trail size on-the-fly

View -> Toolbars -> Control Toolbox -> Spin Button (drag create)

Right click the button and select “Properties”

Change the following:

(Name): Trail_Size

Min: 0

Max: 500

Macro associated to button “Trail Size”

```
Private Sub Trail_Size_Change()
Range("B21") = 10 * Trail_Size.Value
Range(Range("D29").Offset(Range("B21"), 0), _
"L5000").ClearContents
End Sub
```

Just a line break in VBA (the line was split in 2 by using “ _”

	A	B
1	*Initial conditions*	
2	x1	-4
3	y1	0
4	x2	2
5	y2	-2
6	v1x	0
7	v1y	-2
8	v2x	0
9	v2y	1
10	k	30
11	dt	0.05
12		
13	Zoom	4
14	m1	1
15	m2	2
16		
17		
18	Start-Pause	
19		
20	Index	0
21	Trail Size	200

Create button "Start Pause"

This button will start or pause the simulation

On Draw menu: Auto Shapes -> Basic Shapes -> Rounded Rectangle (drag)

On Draw menu: Text box -> Left Click in the Rectangle -> Type "Start-Pause"

Right Click the rectangle -> Assign macro -> Choose "Start-Pause" from menu

Macro associated to button "Start Pause"

```
Public s As Boolean
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Sub Start_Pause()
If s = False Then
s = True
    Do
        DoEvents
        If s = False Then Exit Do
        Range("D28", Range("L28").Offset(Range("B21"), 0)) = _
        Range("D27", Range("L27").Offset(Range("B21"), 0)).Value
    Loop
Else
    s = False
    Exit Sub
End If
End Sub
```

Just a line break in VBA (the line was split in 2 by using " _")

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Observation

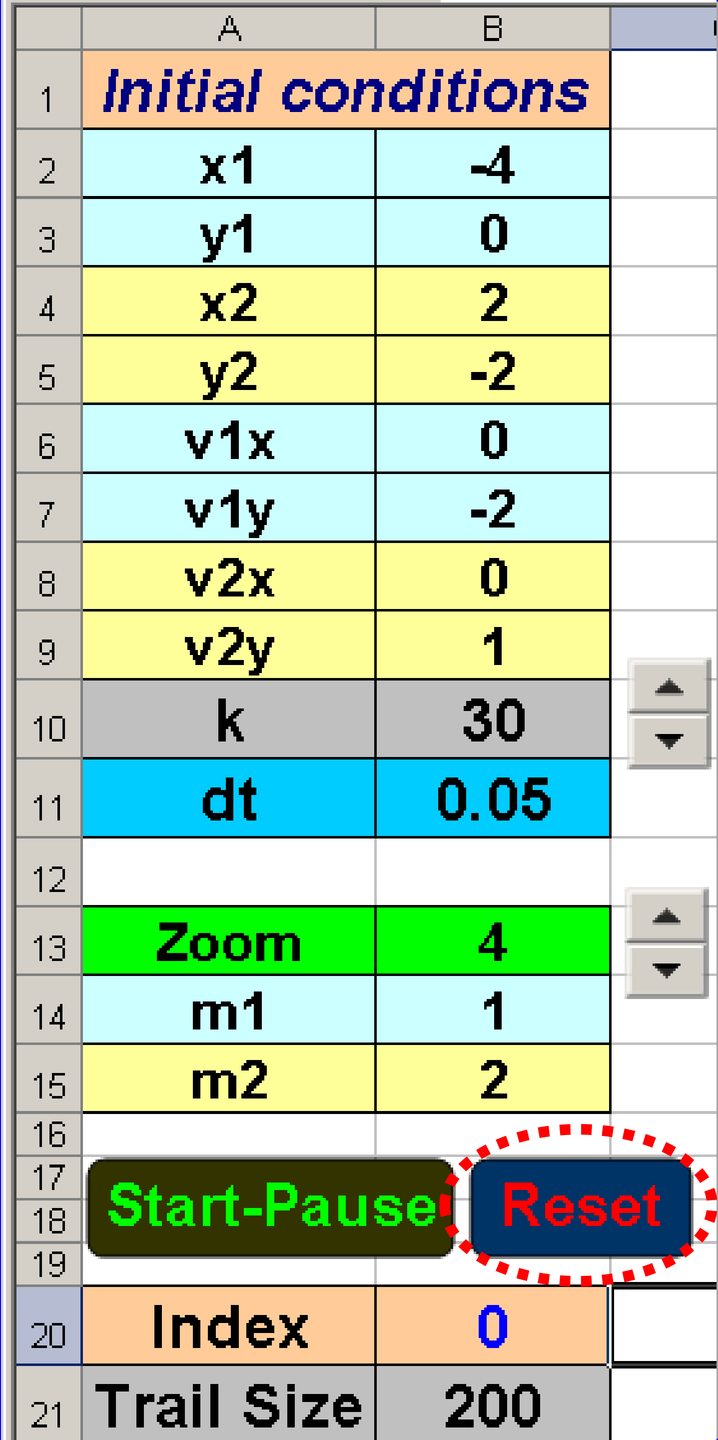
The original line of code didn't give a trail:

```
Range("D28:L28")= Range("D27:L27").Value
```

Was replaced with:

```
Range("D28", Range("L28").Offset(Range("B21"), 0)) = _
Range("D27", Range("L27").Offset(Range("B21"), 0)).Value
```

Which creates a trail with programmable length behind each planet



Create button “Reset”

This button will reset the simulation

On Draw menu: Auto Shapes -> Basic Shapes -> Rounded Rectangle (drag)

On Draw menu: Text box -> Left Click in the Rectangle -> Type “Reset”

Right Click the rectangle -> Assign macro -> Choose “Reset_” from menu

Macro associated to button "Reset"

```
Sub Reset_()
    Range("D28:L5000").ClearContents
    Range("D28") = Range("B6").Value
    Range("E28") = Range("B7").Value
    Range("F28") = Range("B8").Value
    Range("G28") = Range("B9").Value
    Range("H28") = Range("B2").Value
    Range("I28") = Range("B3").Value
    Range("J28") = Range("B4").Value
    Range("K28") = Range("B5").Value
    s = False
End Sub
```

The calculation area

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	A	B	C	D	E	F	G	H	I	J	K	L
24												
25		accelerations			speeds			coordinates				
26		a1x	a1y	v1x	v1y	v2x	v2y	x1	y1	x2	y2	time
27	t[n]	1.89737	-0.6325	0.095	-2.03	-0.05	1.02	-4	-0.1	2	-1.95	0.05
28	t[n-1]											

Cell “B26”: a1x   Cell “C26”: a1y

Cell “A27”: t[n]

Cell “D26”: v1x   Cell “E26”: v1y

Cell “A28”: t[n-1]

Cell “F26”: v2x   Cell “G26”: v2y

Cell “B25”: accelerations

Cell “H26”: x1   Cell “I26”: y1

Cell “E25”: speeds

Cell “J26”: x2   Cell “K26”: y2

Cell “I25”: coordinates

Cell “L26”: time

Just names up to this point !

The calculation area – Active Formulas

	A	B	C	D	E	F	G	H	I	J	K	L
24												
25		accelerations		speeds				coordinates				
26		a1x	a1y	v1x	v1y	v2x	v2y	x1	y1	x2	y2	time
27	t[n]	1.89737	-0.6325	0.095	-2.03	-0.05	1.02	-4	-0.1	2	-1.95	0.05
28	t[n-1]											

Cell “B27”: =B10*B15*B14*(J28-H28)/((J28-H28)^2+(K28-I28)^2)^(3/2)

Cell “C27”: =B10*B15*B14*(K28-I28)/((J28-H28)^2+(K28-I28)^2)^(3/2)

Cell “D27”: =D28+B27*B11/B14

Cell “E27”: =E28+C27*B11/B14

Cell “F27”: =F28-B27*B11/B15

Cell “G27”: =G28-C27*B11/B15

The calculation area – Active Formulas

	A	B	C	D	E	F	G	H	I	J	K	L
24												
25		accelerations		speeds				coordinates				
26		a1x	a1y	v1x	v1y	v2x	v2y	x1	y1	x2	y2	time
27	t[n]	1.89737	-0.6325	0.095	-2.03	-0.05	1.02	-4	-0.1	2	-1.95	0.05
28	t[n-1]											

Cell “H27”: =H28+D27*B11

Cell “I27”: =I28+E27*B11

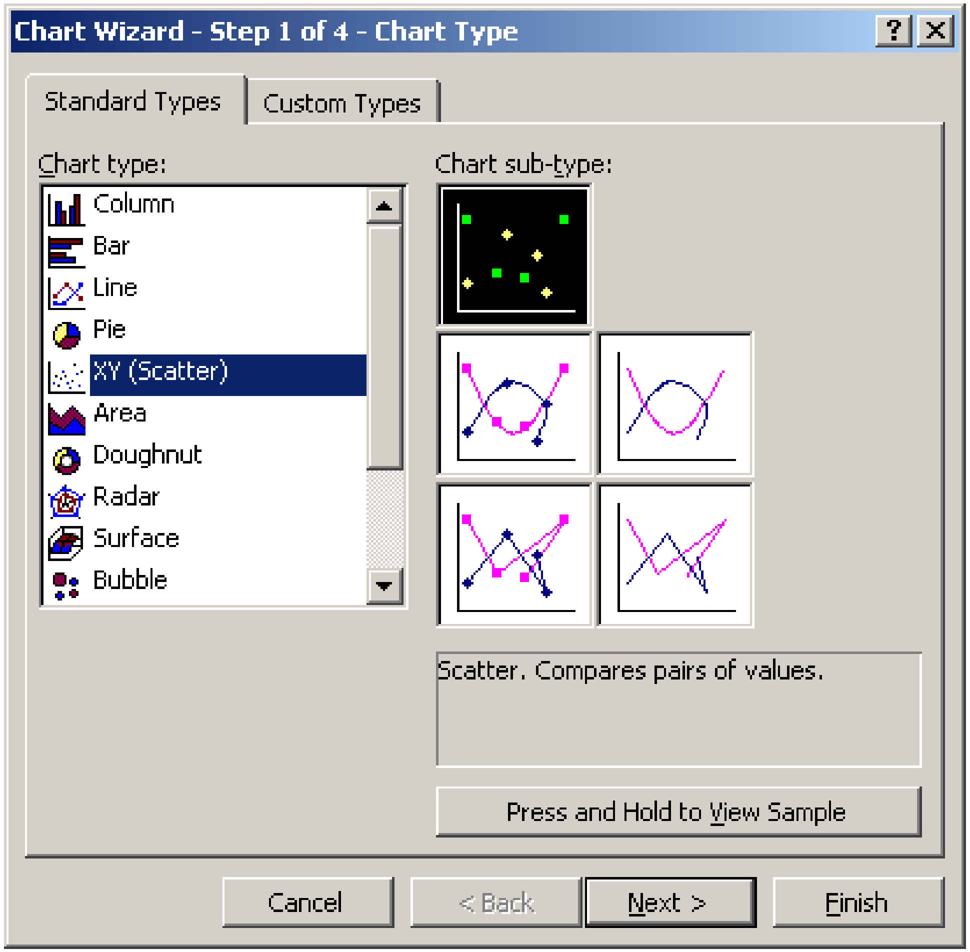
Cell “J27”: =J28+F27*B11

Cell “K27”: =K28+G27*B11

Cell “L27”: =L28+B11

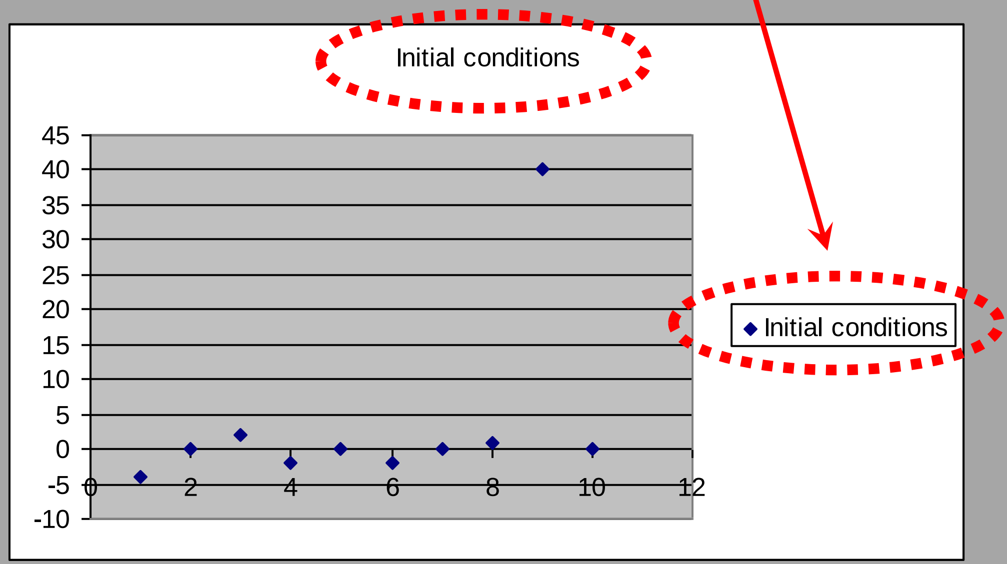
Create the chart

Click on an empty cell -> Insert -> Chart -> -> XY Scatter -> Finish



Create the chart - continuation

Delete Legend and Title



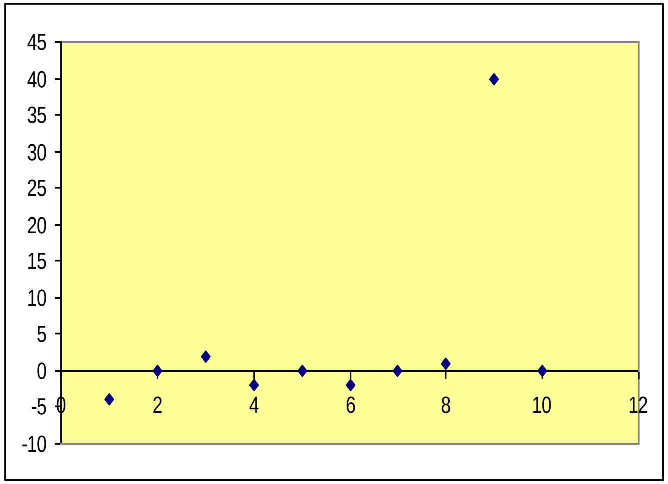
Create the chart - continuation

- Click on any grid lines and delete them then change the charting area color to your taste

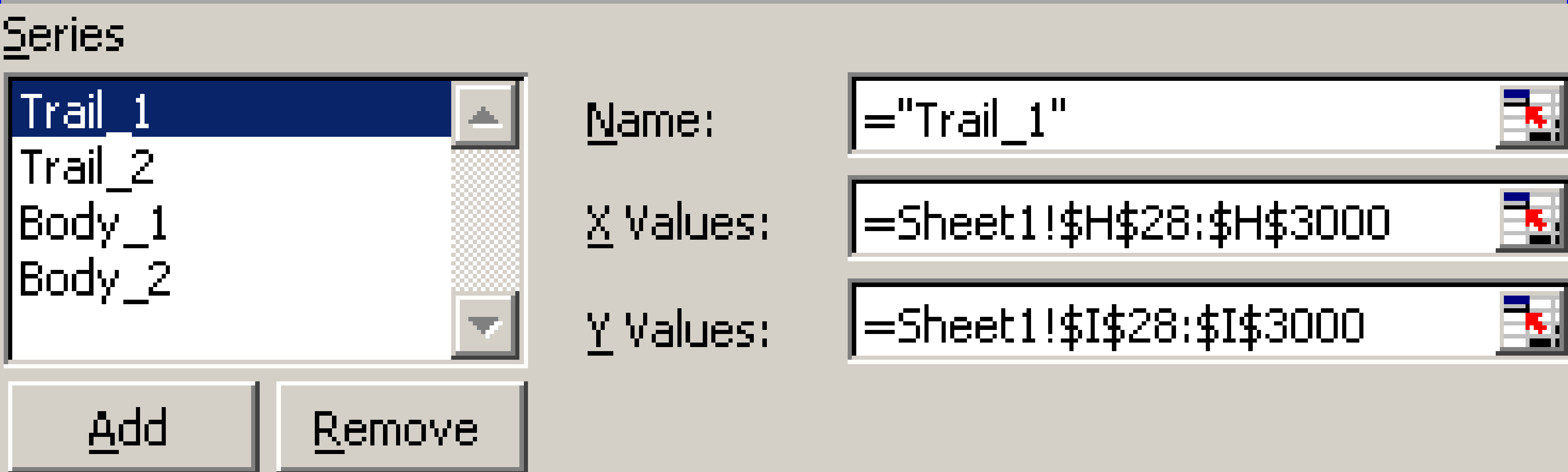
- Change the font on both axes to 1 (using Format Axis)

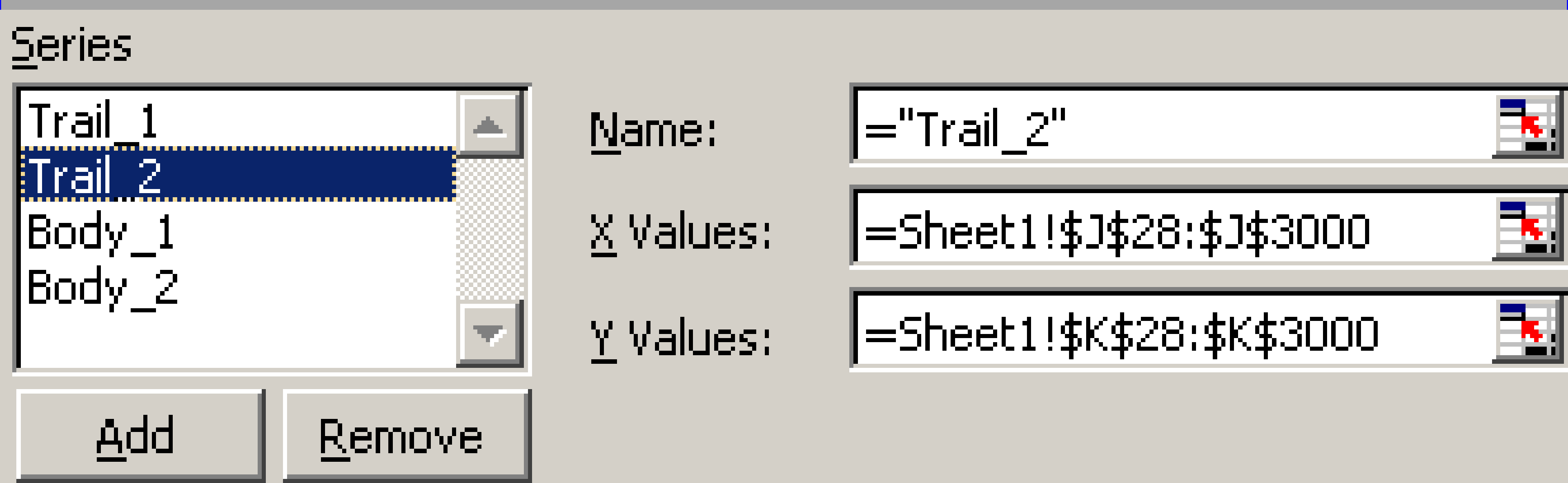
- Click the white area -> Source Data Series

- Remove whatever series is there and Add the following four series (type them in) ->

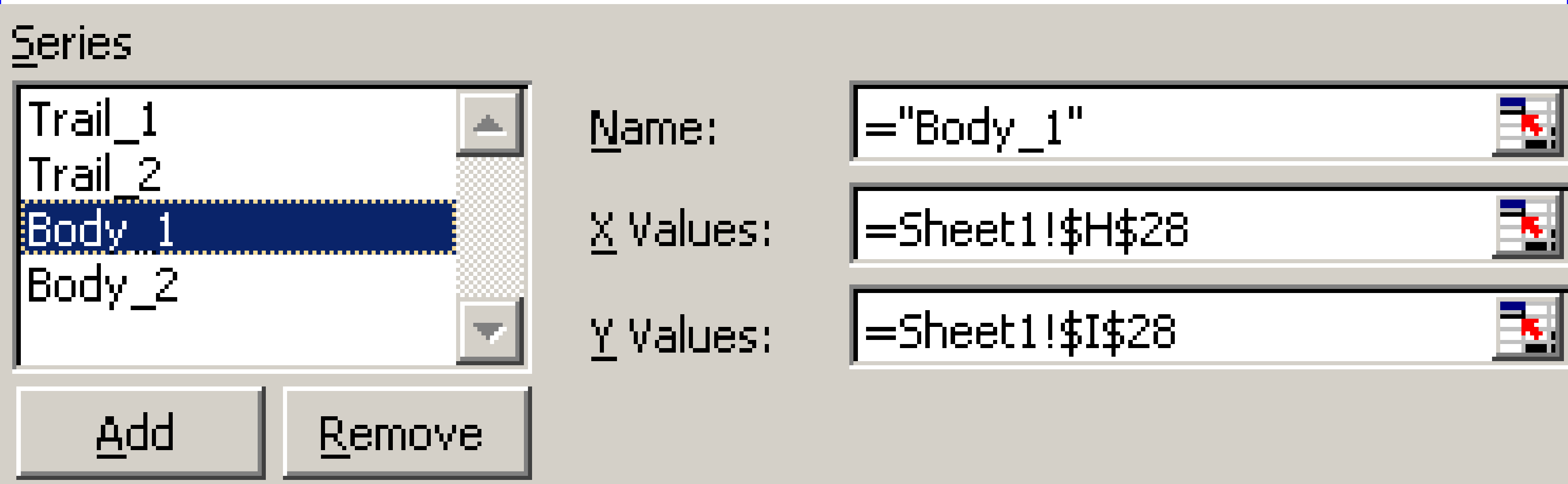


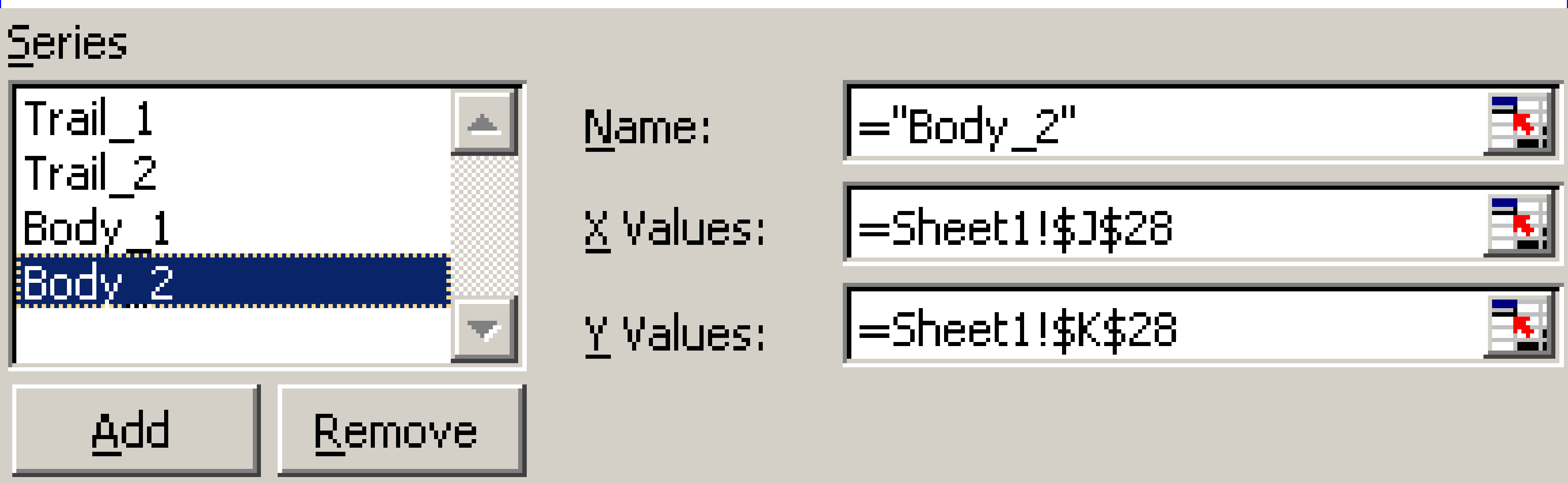
Generate chart data series <excelunusual.com>





Generate chart data series





Create the chart - continuation

In order to have the Zoom macro work right you have to name the chart “Chart 1”

You do this by selecting the chart and running the following macro:

```
Sub rename()
With ActiveChart
.Parent.Name = "Chart 1"
End With
End Sub
```

Finish

Click the button "Reset" and then "Zoom".

This will reset the calculations and resize the chart to the proper zoom level

Click the button "Trail_Size" up and down once

Then click the button "Start-Pause"

Wait few seconds and click "Start-Pause" again

Now go on the chart and adjust the colors and sizes of the bodies and their trails to your liking

Tips

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You can experiment with various combinations of masses and initial conditions

If *dt* is too large de simulation won't converge.

You can change the zoom, trail size and even *k* during the simulation.
Be careful with *k*, change it slowly during the run. If you loose the bodies *Reset* the system and start it with the new *k*

If you get stuck ask for help. In this case or if you just want do avoid the pain of creating one I can send you a copy of the original file.