3D-2D Perspective Mapping in Excel – Part #2

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Introduction:

The first half of this presentation dealt with mapping a 3D scene onto a 2D surface which can be a computer screen, a projection screen or the retina of the eye. By doing so we preserve much of the feel of depth of the scene.

Of course when we look at an picture we get only part of the feel of depth because our both eyes see the same image. Stereoscopic view, refers to a technique for further enhancing the illusion of depth in an image by presenting two offset images separately to each eye of the viewer. We do not deal with this here – at least not yet.

The whole proof of the first half of the presentation was done on a very particular setup when the origin of the object system of coordinates was perfectly centered with the eye-to-center-of-screen, also axis x was parallel to u and z was parallel to v. Though it seems like a particular conversion case, the degree of generalization is still maintained when we can use a secondary object system of coordinates which is translated and rotated from the original object system of coordinate by arbitrary values.

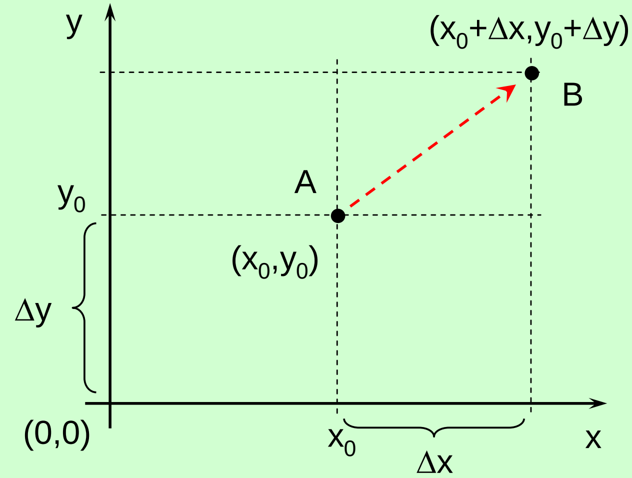
In this presentation we will review the concepts of translation and rotation and then apply them in conjunction with the perspective conversion to achieve a more general three dimensional model in Excel.

Translation

- The following coordinate transformation of point (x_0, y_0, z_0) defines a translation by $(\Delta x, \Delta y, \Delta z)$:

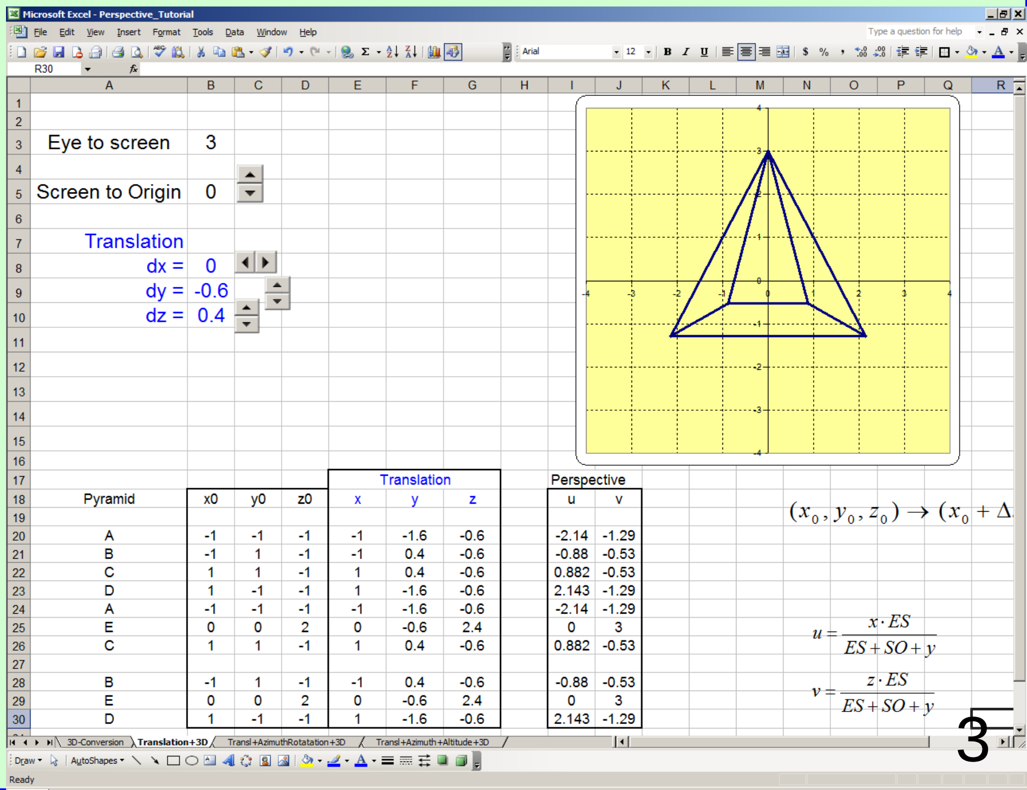
$$(x_0, y_0, z_0) \rightarrow (x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z)$$

Illustration of a 2D translation operation (a 3D translation is harder to draw so I saved some effort)



Translation + 3D Perspective Conversion – *spreadsheet implementation*

- This stage is implemented in a new worksheet named, “Translation+3D”



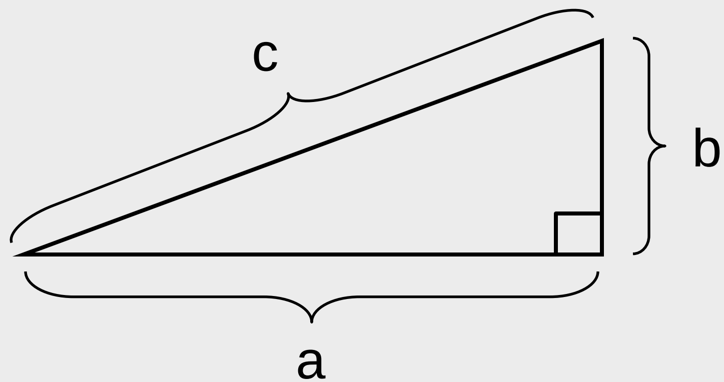
Pyramid	x0	y0	z0	Translation x	y	z	Perspective u	v
A	-1	-1	-1	-1	-1.6	-0.6	-2.14	-1.29
B	-1	1	-1	-1	0.4	-0.6	-0.88	-0.53
C	1	1	-1	1	0.4	-0.6	0.882	-0.53
D	1	-1	-1	1	-1.6	-0.6	2.143	-1.29
A	-1	-1	-1	-1	-1.6	-0.6	-2.14	-1.29
E	0	0	2	0	-0.6	2.4	0	3
C	1	1	-1	1	0.4	-0.6	0.882	-0.53
B	-1	1	-1	-1	0.4	-0.6	-0.88	-0.53
E	0	0	2	0	-0.6	2.4	0	3
D	1	-1	-1	1	-1.6	-0.6	2.143	-1.29

Azimuth rotation

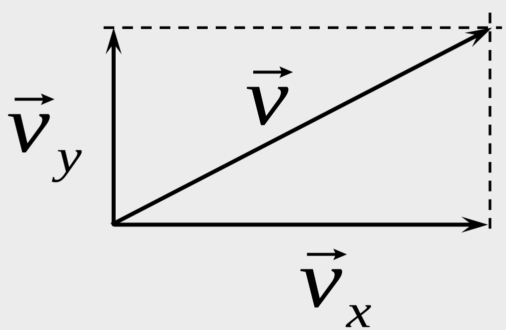
- Azimuth rotation is rotation of an object around the “Z” axis. The z coordinate is unchanged so the rotation calculations are concerned only with the (x,y) coordinates of the position vector. Let’s review the definitions of two basic trigonometric functions on a ***right triangle***:

$$Sin(\alpha) = \frac{b}{c}$$

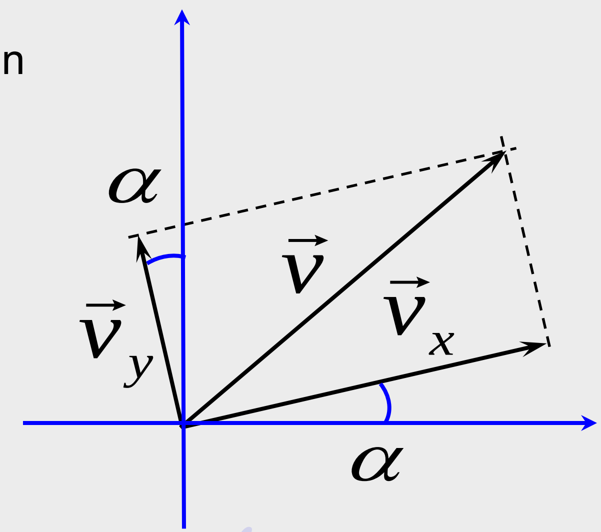
$$\cos(\alpha) = \frac{a}{c}$$



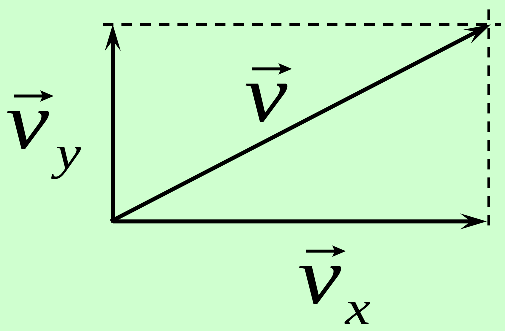
Let’s take a vector with its (x,y) components (again, v is a position vector in the (x,y) plane:



and let’s rotate it by angle α:



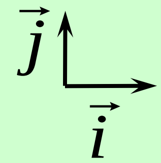
Before the rotation, $\vec{v}_x$ and $\vec{v}_y$ were parallel with the x and y axes respectively and could be expressed function of the axes unit vectors $\vec{i}$, $\vec{j}$:



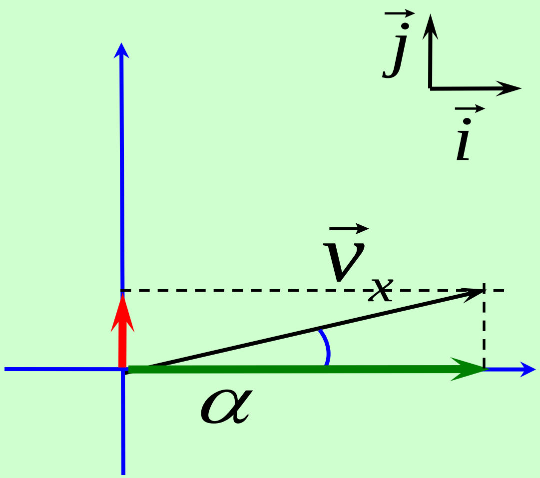
$$\vec{v}_x = \vec{i} \cdot v_x$$

$$\vec{v}_y = \vec{j} \cdot v_y$$

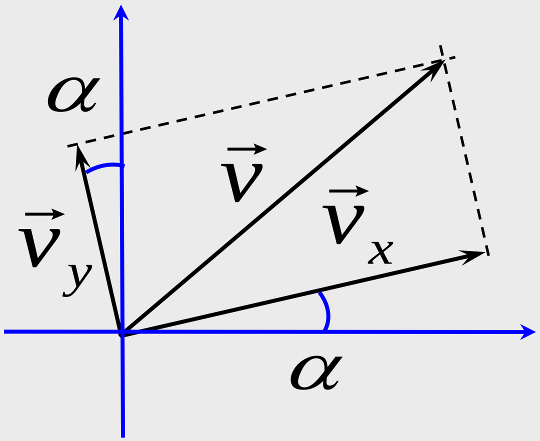
Where $\vec{i}$ and $\vec{j}$ are the x and y unit vectors respectively

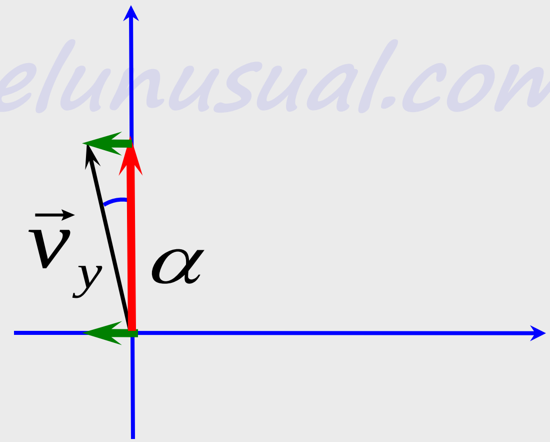


After the rotation, $\vec{v}_x$ and $\vec{v}_y$ are no longer parallel with the x and y axes respectively and can be expressed function of the axes unit vectors $\vec{i}$, $\vec{j}$:



$$\vec{v}_x = \vec{i} \cdot \cos(\alpha) \cdot v_x + \vec{j} \cdot \sin(\alpha) \cdot v_x$$





$$\vec{v}_y = -\vec{i}\cdot\sin(\alpha)\cdot v_y + \vec{j}\cdot\cos(\alpha)\cdot v_y$$

We can express $\vec{v}$ as the vector sum of $\vec{v}_x$ and $\vec{v}_y$

$$\vec{v} = \vec{v}_x + \vec{v}_y =$$

$$= \vec{i}\cdot\cos(\alpha)\cdot v_x + \vec{j}\cdot\sin(\alpha)\cdot v_x - \vec{i}\cdot\sin(\alpha)\cdot v_y + \vec{j}\cdot\cos(\alpha)\cdot v_y$$

Grouping around $\vec{i}$ and $\vec{j}$ results in:

$$\vec{v} = \vec{i}\cdot[\cos(\alpha)\cdot v_x - \sin(\alpha)\cdot v_y] + \vec{j}\cdot[\sin(\alpha)\cdot v_x + \cdot\cos(\alpha)\cdot v_y]$$

Calling v_x' and v_y' the new coordinates of the rotated point around origin we obtain the following formulas:

$$\begin{cases} v_x' = \cos(\alpha)\cdot v_x - \sin(\alpha)\cdot v_y \\ v_y' = \sin(\alpha)\cdot v_x + \cos(\alpha)\cdot v_y \end{cases}$$

(where α is the rotation angle and v_x and v_y are the original coordinates before the rotation)

Don't memorize but be able to derive this at any time!

Sometimes people like to put this in matrix form:

$$\begin{pmatrix} v_x' \\ v_y' \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

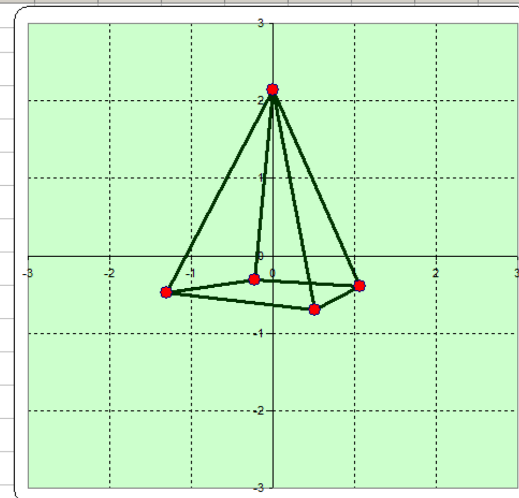
Let's see how the three transformations studied so far (translation, azimuth rotation and 3D-2D perspective) can be incorporated in the same worksheet =>>>

Translation + Azimuth Rotation + 3D Perspective – *spreadsheet implementation*

- This stage is implemented in a new worksheet named, “Translation+AzimuthRotation+3D”

Microsoft Excel - Perspective_Tutorial

Eye to screen	3
Screen to Origin	0.5
Translation	
dx =	0
dy =	0
dz =	0.5
Azim. Rotation [deg]	330



				Translation			Azimuth Rotation			Perspective	
Pyramid	x0	y0	z0	x	y	z	x'	y'	z'	u	v
A	-1	-1	-1	-1	-1	-0.5	-1.37	-0.37	-0.5	-1.31	-0.48
B	-1	1	-1	-1	1	-0.5	-0.37	1.366	-0.5	-0.23	-0.31
C	1	1	-1	1	1	-0.5	1.366	0.366	-0.5	1.06	-0.39
D	1	-1	-1	1	-1	-0.5	0.366	-1.37	-0.5	0.515	-0.7
A	-1	-1	-1	-1	-1	-0.5	-1.37	-0.37	-0.5	-1.31	-0.48
E	0	0	2	0	0	2.5	0	0	2.5	0	2.143
C	1	1	-1	1	1	-0.5	1.366	0.366	-0.5	1.06	-0.39
B	-1	1	-1	-1	1	-0.5	-0.37	1.366	-0.5	-0.23	-0.31
E	0	0	2	0	0	2.5	0	0	2.5	0	2.143
D	1	-1	-1	1	-1	-0.5	0.366	-1.37	-0.5	0.515	-0.7

$$(x_0, y_0, z_0) \rightarrow$$

$$v_x' = \cos(\alpha)\cdot v_x$$

$$v_y' = \sin(\alpha)\cdot v_x$$

$$u = \frac{x \cdot ES}{ES + SO + y}$$

$$v = \frac{z \cdot ES}{ES + SO + y}$$

3D-Conversion / Translation+3D / Transl+AzimuthRotatation+3D / Transl+Azimuth+Altitude+3D

Altitude rotation

- Altitude rotation is the rotation of an object around the "X" axis. The x coordinate is unchanged so the rotation calculations are concerned only with the (y,z) coordinates of the position vector. By the same logic followed during azimuth derivation we have:

$$\begin{cases} v_y'' = \cos(\beta)\cdot v'_y - \sin(\beta)\cdot v'_z \\ v_z'' = \sin(\beta)\cdot v'_y + \cos(\beta)\cdot v'_z \end{cases}$$

(where β is the rotation angle and v'_y and v'_z are the original coordinates before the rotation)

Translation + Azimuth Rotation + Altitude Rotation + 3D conversion – *implementation*

- This stage is implemented in a new worksheet named, "Transl+Azimuth+Altitude+3D"
- All the translation and rotation parameters are adjusted by spinner buttons
- Verifying the implementation is left as an exercise to the reader