

Projectile Motion Tutorial #5

- a custom VBA projectile dynamics function returning a 2D array

The model from the last presentation - 2D projectile motion with air drag - had only two significant (large) formulas.

In the current, we already got all the modeling concentrated in just two functions (the coordinate functions) therefore we will write a custom VBA function (returning a 2D array).

This process of concentrating the calculations and writing custom functions for those calculations will be extremely useful later for building complex models.

The coordinate formulas and notations:

- Using the notations to the right we can write the custom VBA function below.

- The custom function uses three intermediate variables declared inside the function body and the output is a horizontal array of size 2 x 1.

$$x_0 = x_1 + (x_1 - x_2) \cdot \left(1 - \frac{\overbrace{\rho \cdot A \cdot Cx \cdot \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}^{\text{drag_product}}}{2 \cdot m} \right)$$
$$y_0 = y_1 + (y_1 - y_2) \cdot \left(1 - \frac{\rho \cdot A \cdot Cx \cdot \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{2 \cdot m} \right) - g \cdot \Delta t^2$$

Diagram illustrating the formulas and their components:

- x_0 is calculated from x_1 and x_2 .
- y_0 is calculated from y_1 and y_2 .
- The drag product is defined as $\rho \cdot A \cdot Cx \cdot \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
- The mass is $2 \cdot m$.
- The acceleration due to gravity is g .
- The time step is Δt .

The macro:

```
Function Projectile(x_1, x_2, y_1, y_2, mass, area, air_density, cx, g, time_step)
Dim drag_product, x_0, y_0 As Double
drag_product = air_density * area * cx * Sqr((x_1 - x_2) ^ 2 + (y_1 - y_2) ^ 2) / (2 * mass)
x_0 = x_1 + (x_1 - x_2) * (1 - drag_product)
y_0 = y_1 + (y_1 - y_2) * (1 - drag_product) - g * time_step ^ 2
Projectile = Array(x_0, y_0)
End Function
```

Spreadsheet usage:

- The output of the “Projectile()” function is a 2X1 array.
- If you just enter the function “Projectile()” in your worksheet in the normal way, in a single cell, it will return a single number. In this case, that single number will be the x coordinate, which is the first element of the array. If you want to see both coordinates in neighboring cells, you have to enter the function as an array, which means select a range of two neighboring cells in the same row, enter the formula and then hit Ctrl-Shift-Enter to create the array function.
- If you want to copy, paste or cut the function you need to select the cell pair (the whole function to do the operation)
- If you need to change one or several arguments of the “Projectile()” function you do it the regular way by selecting one cell than typing or dragging the argument on the worksheet, but after you finish you need again to hit Ctrl-Shift-Enter for the changes to take effect. If you just hit Enter you get the error message: “You cannot change part of an array”.

Excel implementation:

- Copy the last worksheet into a new one called “Tutorial_7”
- Reassign the “Fire” macro corresponding to this last worksheet to the button contained in the same worksheet
- Select range D27:E27 and type:
“=Projectile(D26,D25,E26,E25,B\$7,B\$3,B\$1,B\$5,9.81,B\$16)”
- After you typed that, hit Ctrl-Shift-Enter
- Copy D27:E27 down to row 2100

	A	B	C	D	E
15					
16	Time Step [sec]	0.02			
17					
18					
19			Fire		
20	Index	0			
21					
22					
23					
24				x	y
25	Initial Conditions	v_x	v_y	-0.2	6.65
26		10	17.3205	0	7
27				1,B\$16)	7.34
28				0.39111	7.67
29				0.58245	7.99
30				0.77112	8.3