A casual approach to numerical modeling - part #5

a Spring-Mass-Damper-System - condensing the calculations

- Up until now our table of formulas contained three columns (accelerations, velocities and coordinates). We would like to simplify that and keep everything function only of the coordinates. It's true that the coordinate formula will increase in complexity but it is advantageous, especially for complex systems to have fewer cells with calculations. Eventually we could create custom VBA formulas for those few cells.

From tutorial # 2 we have the formulas below:

$$x_{current} = x_{previous} + v_{current} \cdot dt$$

$$v_{current} = v_{previous} + a_{current} \cdot dt$$

$$a_{current} = -\left(k \cdot x_{previous} + 2 \cdot DR \cdot v_{previous} \cdot \sqrt{m \cdot k}\right) / m$$

Let's use the following convention for the indices:

$$current = 0$$

$$previous = -1$$

$$previous(previous) = -2$$

$$\begin{cases} x_0 = x_{-1} + v_0 \cdot dt \\ v_0 = v_{-1} + a_0 \cdot dt \\ a_0 = -\left(k \cdot x_{-1} + 2 \cdot DR \cdot v_{-1} \cdot \sqrt{m \cdot k}\right) / m \end{cases}$$

Eliminating the current acceleration and velocity we get the coordinate formula below:

$$\boxed{x_0 = x_{-1} + v_{-1} \cdot dt - \left(k \cdot x_{-1} + 2 \cdot DR \cdot v_{-1} \cdot \sqrt{m \cdot k}\right) \cdot dt^2 / m}$$

By George Lungu

- Using the definition of speed applied to the previous time step (v_{-1}):

$$v_{-1} = (x_{-1} + x_{-2}) / dt$$

- We arrive to a coordinate formula dependent only on the coordinate history and the input constants (m, k, DR, dt):

$$x_0 = 2 \cdot x_{-1} - x_{-2} - \frac{\left(k \cdot x_{-1} \cdot dt^2 + 2 \cdot DR \cdot (x_{-1} - x_{-2}) \cdot \sqrt{m \cdot k} \cdot dt\right)}{m}$$

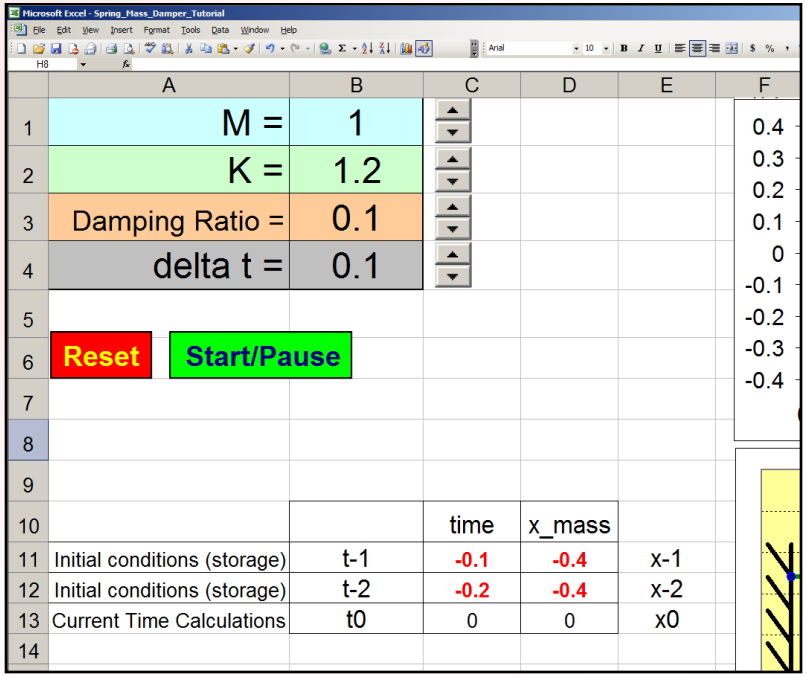
- After basic manipulations we arrive to the following formula which we can use in the spreadsheet and also for creating a custom VBA coordinate function:

$$\boxed{x_0 = x_{-1} \cdot \left(1 - \frac{k}{m} \cdot dt^2\right) + (x_{-1} - x_{-2}) \cdot \left(1 - 2 \cdot DR \cdot \sqrt{\frac{k}{m}} \cdot dt\right)}$$

- We can easily see that now the coordinate function is dependent only on the coordinates of the previous two time steps and the input constants m, k, DR, dt. With this formula we can use only one column of calculations/history (x) instead of three (a, v, x)

Spreadsheet implementation:

- Copy the last worksheet, and rename the copy "Tutorial_5"
- Right click each of the two buttons and assign the macro corresponding to the new worksheet (Tutorial_5). This is necessary because whenever one copies a worksheet, the new buttons have still attached the old macros from the "mother worksheet"
- "Reset" the model by clicking the Reset button, this will delete the history in the calculation area corresponding to the active worksheet
- Delete everything in the area C7:E10 and add the following labels following the example in the snapshoot below.



-Range C11:D12 represents the initial conditions and range C13:D13 contains the current step calculations (the active x0 formula will be placed in cell D13)

- Fill in the initial conditions: C11: "=-B4", C12: "=-2*B4", D11: "=-0.4", D12: "=-0.4"
- You can choose the initial coordinates different than -0.4 if you wish
- Type in the current coordinate formula:

D13: "=D14*(1-B$2*B$4^2/B$1)+(D14-D15)*(1-2*B$3*B$4*SQRT(B$2/B$1))"

which is the formula to the right => $x_0 = x_{-1} \cdot \left(1 - \frac{k}{m} \cdot dt^2\right) + \left(x_{-1} - x_{-2}\right) \cdot \left(1 - 2 \cdot DR \cdot \sqrt{\frac{k}{m}} \cdot dt\right)$

The "reset" macro:

- At the beginning of this tutorial we took the acceleration, speed and coordinate and expressed them function of past coordinates. The initial conditions have to reflect that, namely they have to be expressed only function of past coordinates.

- This is a modified version of the previous "reset" macro. After clearing the history, and setting the current time to zero, this macro pastes the initial conditions (x_{-1} and x_{-2}) in the rows just under the active coordinate calculation (x_0)

```
Sub reset()
DoEvents
Range("C13") = 0
Range("C14:D1014").Clear
Range("C14:D15") = Range("C11:D12").Value
End Sub
```

The "StartStop" macro:

- This macro uses a Boolean variable (RunSim) to start or stop a conditional "Do" loop. If the macro is running then RunSim=True, hitting the macro button again will change RunSim to false, stop the conditional "Do" loop therefore stop the simulation. If the macro is stopped the reverse happens and the "Do" loop is initiated.

- The Do loop within the macro shifts all the historical coordinate and time information back in time hence advancing the simulation. It achieves this by a copy-and-paste-below type of operation, done every time step right after the new current coordinate is computed. The historical, while not necessary for calculations is still recorded for charting purposes.

```
Sub StartStop()
   RunSim = Not (RunSim)
   Do While RunSim = True And [C13] < 31
   DoEvents
   Range("C14:D1014") = Range("C13:D1013").Value
   Range("C13") = Range("C13") + Range("B4")
   DoEvents
   Loop
End Sub
```

Let's see how the model works:

9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0	-0.3952	x0
14			-0.1	-0.4	
15			-0.2	-0.4	
16					
17					
18					
19	Right after clicking "Reset"				
20					

Right after reset: The reset macro cleared all history below current time (row 13) then pasted the initial values in the most recent history place (range C14:D15). The "current coordinate" formula evaluates a result based on the values in the range D14:D15.

9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0.1	-0.38576	x0
14			0	-0.3952	
15			-0.1	-0.4	
16			-0.2	-0.4	
17					
18					
19	Right after clicking "StartStop"				
20					

First Do loop cycle: The macro copies all historical data from range C13:D2013 and shifts it down one row (one time step), then it increments the current time by Δt (the time step) to 0.1 sec. After that the spreadsheet calculates a new coordinate using the values in the range D14:D15.

9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0.2	-0.3719	x0
14			0.1	-0.38576	
15			0	-0.3952	
16			-0.1	-0.4	
17			-0.2	-0.4	
18					
19					
20					

Second Do loop cycle: The StartStop macro copies all historical data from range C13:D2013 and shifts it down one row (one time step). Then macro increments the current time by Δt (the time step) to 0.2 sec. After that the spreadsheet calculates a new coordinate using the values in the range D14:D15.

9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0.3	-0.35388	x0
14			0.2	-0.3719	
15			0.1	-0.38576	
16			0	-0.3952	
17			-0.1	-0.4	
18			-0.2	-0.4	
19					
20					

Third Do loop cycle: The StartStop macro copies all historical data from range C13:D2013 and shifts it down one row (one time step). Then macro increments the current time by Δt (the time step) to 0.3 sec. After that the spreadsheet calculates a new coordinate using the values in the range D14:D15.

9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0.4	-0.33201	x0
14			0.3	-0.35388	
15			0.2	-0.3719	
16			0.1	-0.38576	
17			0	-0.3952	
18			-0.1	-0.4	
19			-0.2	-0.4	
20					

Fourth Do loop cycle: The StartStop macro copies all historical data from range C13:D2013 and shifts it down one row (one time step). Then macro increments the current time by Δt (the time step) to 0.4 sec. After that the spreadsheet calculates a new coordinate using the values in the range D14:D15.

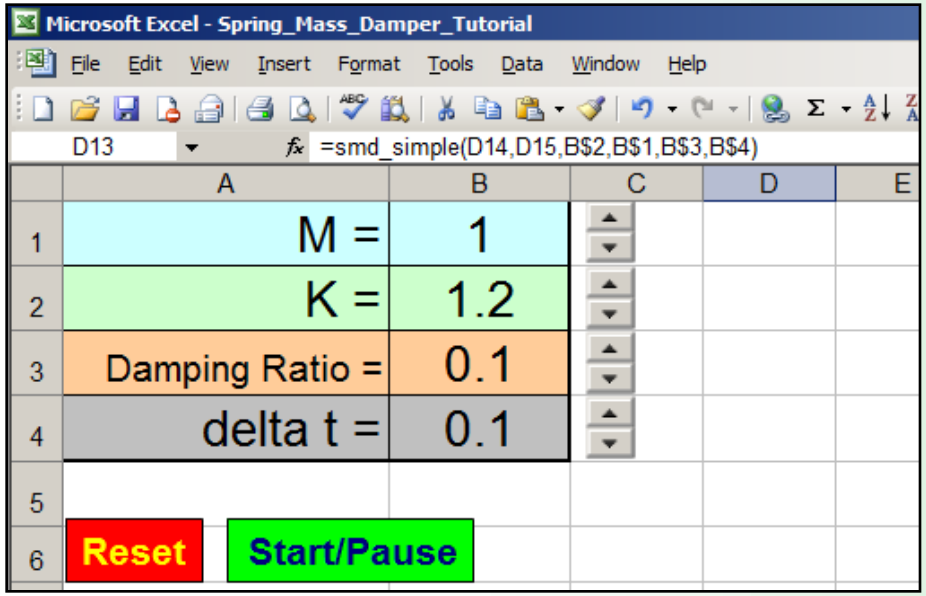
9					
10			time	x_mass	
11	Initial conditions (storage)	t-1	-0.1	-0.4	x-1
12	Initial conditions (storage)	t-2	-0.2	-0.4	x-2
13	Current Time Calculations	t0	0.5	-0.30664	x0
14			0.4	-0.33201	
15			0.3	-0.35388	
16			0.2	-0.3719	
17			0.1	-0.38576	Etc, etc
18			0	-0.3952	
19			-0.1	-0.4	
20			-0.2	-0.4	

Fifth Do loop cycle: The StartStop macro copies all historical data from range C13:D2013 and shifts it down one row (one time step). Then macro increments the current time by Δt (the time step) to 0.4 sec. After that the spreadsheet calculates a new coordinate using the values in the range D14:D15.

A custom VBA function:

- Copy the last worksheet, and rename the copy "Tutorial_5_Custom_Function"
- Right click each of the two buttons and assign the macro corresponding to the new worksheet (Tutorial_5). This is necessary because whenever one copies a worksheet, the new buttons have still attached the old macros from the "mother worksheet"
- Reset the model by clicking the "Reset" button, this will delete the history in the calculation area corresponding to the active worksheet
- Insert a module and inside write the following VBA user defined function.

```
Function smd_simple(x_1, x_2, k, m, dr, dt) As Double
Dim kmdt As Double
kmdt = k * dt ^ 2 / m
SMD_simple = x_1 * (1 - kmdt) + (x_1 - x_2) * (1 - 2 * dr * Sqr(kmdt))
End Function
```



Insert the formula in the worksheet:

D13: "=smd_simple(D14,D15,B$2,B$1,B$3,B$4)"