Animated heat transfer modeling for the average Joe – part #2

- This is a continuation of the first part of the beginner series of tutorials in heat transfer modeling. The first part introduced the reader to the concept of heat capacity (being analogous to the electrical capacity).

- This section continues with the concept of heat conductance which is analogous to the electrical conductance. A law analogous to Ohm's law governs the conduction process.

- Toward the end, the principle of using both concepts together (heat storage and heat conduction) in a finite element type of scheme will be explained. Understanding this concept well will later allow the reader to numerically model heat transfer with ease on complex 3D shapes with minimum experience or mathematical knowledge.

<www.excelunusual.com>

by George Lungu

Review of the basic heat storage principles and equation:

(IEEIKS = If Everything Else Is Kept the Same)

1. IEEIKS a body stores heat proportional to its mass.
2. IEEIKS a body stores heat in relation to the substance it is made from.
3. IEEIKS the net amount of heat a body exchanges with the environment in a certain time interval is proportional to the net temperature change the same body experiences during that time interval.

The previously mentioned principles are incorporated in the heat storage equation. "ΔQ" is the variation of the stored heat [Joules], "m" is the mass of the object [Kg], "c" is called specific heat and it's a material specific property [J/Kg*K] and "ΔT" is the temperature variation [K].

$$\Delta Q = c \cdot m \cdot \Delta T$$

The product of mass and specific heat is known as thermal or heat capacity :

$$C_{thermal} = c \cdot m$$

The heat storage equation becomes:

$$dQ = C_{thermal} \cdot dT$$

The basic heat transport principle and equation:

1. The heat flow rate between two bodies is directly proportional to the difference in temperature between the bodies with a constant of proportionality known as thermal conductance.

- The heat flow rate is the amount of heat moved per unit time.
- In this care we expressed the heat flow from body#1 to body#2
- Looking at the formula we can say that that the thermal conductance is expressed in W/K (or J/s*K).

$$\frac{dQ}{dt} = G \cdot (T_1 - T_2)$$

Heat transport equation - analogy with Ohm's law in electricity and water flow

- The heat transport equation says that the speed of movement of heat (i.e. power) between two sections of a heat conductor is proportional to the difference in temperature between the sections. The constant of proportionality is the thermal conductance expressed in [W/K].

$$Power = \frac{dQ}{dt} = G_{thermal} \cdot (T_2 - T_1)$$

- This is just like Ohm's law in electricity which says that the speed of movement of electric charge (i.e. electric current) in a wire between two points is proportional to the difference in voltage between the points, the constant of proportionality being the electric conductance [A/V – Siemens].

$$Current\_Intensity = \frac{dq}{dt} = G_{electric} \cdot (V_2 - V_1)$$

- We can also use Ohm's law on a water stream or in a system of hydraulic pipes. Ohm's law would say that the flow of fluid (fluid mass/unit time) through a certain pipe between two sections is equal to the product of hydraulic conductance times the difference in pressure between the sections.

$$Water\_Flow = \frac{dm}{dt} = G_{hydraulic} \cdot (P_2 - P_1)$$

More about thermal conductance of a linear uniform "heat pipe":

- Just like in electricity a wider connection (larger sectional area of the heat conductor) between the bodies will result in easier heat transport therefore a higher conductance.
- Just like in electricity a longer connection (longer heat conductor) between the bodies will result in slower heat transport therefore a lower conductance.
- Just like in electricity the material of the heat conductor will affect the heat transport (metals for instance will facilitate the heat transport whereas wood for instance will slow down the heat flow), therefore the conductance will have a factor named conductivity (k) which is material specific.

$$G_{thermal} = k_{thermal}\frac{A}{x} \qquad G_{electrical} = k_{electrical}\frac{A}{x} \qquad G_{hydraulic} = k_{fluid}\frac{A}{x}$$

- Of course the last three relationships apply for the particular case of a uniform wire or a uniform bar (heat or water pipe) but these relationships are very good for intuitively understanding the parameters involved in process of heat conduction.
- Most importantly in the case of a randomly shaped 3D objects, before using numerical methods we will divide the object in regular elements (little parallelepipeds) that you can call bricks or French fries if you wish, on which the simplest formulas (including the above ones) apply.
- Whereas the thermal conductance will be expressed in [W/K - Watts/Kelvin] the thermal conductivity will be expressed in units of [W/K*m].

Let's look at the storage-transport (storage-transfer) pair:

$$dQ = c \cdot m \cdot dT$$

$$\frac{dQ}{dt} = G \cdot (T_1 - T_2)$$

- The first is the storage formula and characterizes the behavior of a barrel, capacitor, bag pipe or trash can. In our case the medium sloshed around is heat but it could be electric charge, water, hash browns or chicken wings if you like. I turned "Δ" into a "d" but don't take it too hard. "d" is a very small "Δ". Use what you like and don't let the whiteboard scribblers intimidate you.

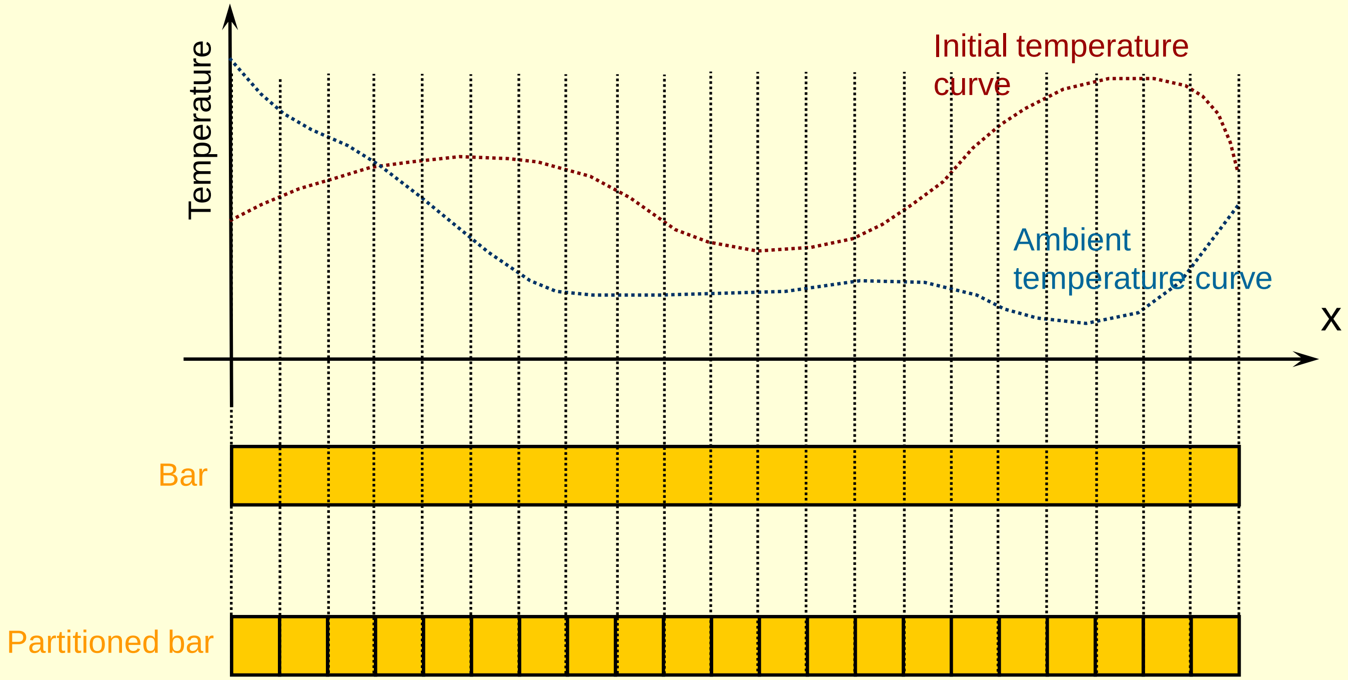
-The second is the transfer or transport formula and refers to "heat conductor" feeding the body or the pipe feeding the trash can. In electricity is called Ohm's law. I could have used ΔT or dT but that could be misleading. Whereas in the storage formula dT or ΔT are always legitimate options since they pertain to a variation of the same value (the body temperature), in the second (transport) formula T1-T2 refers to the difference between temperatures of two neighbors interacting with each other. The difference could be large and sometimes stay large no matter how finely we adjust the simulation step. That's not always the case and you could replace the T1-T2 with dT, just be aware of the difference between a dT in the storage formula and a dT in the transport formula if you see the transport formula written with dT.

- Somebody misinformed could use dT in both and do an algebraic elimination which is a mistake.

- In short the dT in the storage formula is temperature variation in time but in the same point of space and dT in the transport formula is a temperature difference between points and in the case of finite element type of method it depends on how we partitioned our complex object in elements.

Starting the model:

- We will model a linear (1D), uniform bar with a given initial profile of temperature. Heat transfer by conduction will occur within the bar and also the bar will be changing heat by conduction with the ambient whose temperature is characterized by a different temperature map than the initial temperature map of the bar. The ambient has infinite thermal inertia.



- Above is a sketch of the bar. The bar is being partitioned in 21 equal length segments. On the top chart there are two curves: the initial temperature and the ambient temperature. The bar will start from a temperature configuration identical with the brown curve and slowly drift close to the ambient curve. The higher the bar-ambient conductance the closer to the ambient curve the temperature curve of the bar will settle.