

A Fourier Transform Model in Excel #3

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- This is a tutorial about the implementation of a Fourier transform in Excel. The first part went over adjustments that make the general Fourier transform formula applicable on real time sampled signals with a finite number of samples.
 - This second part dealt with the Excel implementation of the input signals.
 - This third part is concerned with implementation of the Fourier transform formulas (real, imaginary, magnitude and phase), the structure of the macros driving the model and the charting of the Fourier transform components

A quick review of the formulas:

The Fourier transform formula derived in the first section of the tutorial is:

$$G(f) \approx \underbrace{h \cdot \sum_0^N g(n \cdot h) \cdot \cos(2\pi \cdot f \cdot n \cdot h)}_{\text{Real part - Re(G(f))}} + j \cdot \left[\underbrace{-h \cdot \sum_0^N g(n \cdot h) \cdot \sin(2\pi \cdot f \cdot n \cdot h)}_{\text{Imaginary part - Im(G(f))}} \right]$$

Rewriting the real part and imaginary part separately we get:

$$\begin{aligned} \text{Re}[G(f)] &\approx \underline{h} \cdot \sum_0^N \underline{g(n \cdot h)} \cdot \underline{\cos(2\pi \cdot f \cdot n \cdot h)} \\ \text{Im}[G(f)] &\approx \underline{-h} \cdot \sum_0^N \underline{g(n \cdot h)} \cdot \underline{\sin(2\pi \cdot f \cdot n \cdot h)} \end{aligned}$$

Underlined with green are quantities already existent in the worksheet. Underlined with red are quantities needed to be further calculated in the worksheet.

- “ h ” is an input parameter situated in cell B11, N is 4999 and it is the height of the calculation tables
- $t = nh$ is the sampled time series and it is located in range A41:A5040
- $g(nh)$ for $n=0,1,2,\dots,4999$ is the sampled real input function and it’s located in range B41:B5040

Implementation of the active formulas : We will set up the spreadsheet to calculate the real and imaginary parts of the FF for only one frequency point, then using a special macro we will change the frequency incrementally in a loop and record the calculated Fourier transform in each of the frequency points from the start frequency to the end frequency.

- Recognizing the fact that we need to estimate the Fourier transform in a series of frequency points we have to replace the frequency “f” with a sampled series of frequencies.
- Since we set up the initial conditions to contain f_{start} , f_{stop} and the number of frequency points (#points), the frequency can be expressed as: $f = f_{start} + m \cdot \Delta f$, where $\Delta f = (f_{stop} - f_{start}) / (\#points)$ and m is the index# (cell B4).
- In this case the real and imaginary parts of the Fourier transform become:

$$\underline{\text{Re}[G(f_{start} + m \cdot \Delta f)]} \approx \underline{h} \cdot \sum_0^N \underline{g(n \cdot h)} \cdot \cos[2\pi \cdot \underline{(f_{start} + m \cdot \Delta f)} \cdot \underline{n \cdot h}]$$

$$\underline{\text{Im}[G(f_{start} + m \cdot \Delta f)]} \approx \underline{-h} \cdot \sum_0^N \underline{g(n \cdot h)} \cdot \sin[2\pi \cdot \underline{(f_{start} + m \cdot \Delta f)} \cdot \underline{n \cdot h}]$$

Let's see how to calculate the remaining parts in the formulas (red underline):

$(f_{start} + m \cdot \Delta f)$ The frequency will be calculated in cell E41: “=B1+B4*(B2-B1)/B3”

$g(n \cdot h) \cdot \cos[2\pi \cdot (f_{start} + m \cdot \Delta f) \cdot n \cdot h]$ this value will be calculated for n=0,1,2,..., 4999 in range C41:C5040;
- cell C41: “=B41*COS(2*PI()*E\$41*A41)” copy C41 down to C5040

$g(n \cdot h) \cdot \sin[2\pi \cdot (f_{start} + m \cdot \Delta f) \cdot n \cdot h]$ this value will be calculated for n=0,1,2,..., 4999 in range D41:D5040;
- cell D41: “=B41*SIN(2*PI()*E\$41*A41)” copy D41 down to D5040

$\text{Re}[G(f_{start} + m \cdot \Delta f)]$ this value will be calculated in cell F41: “=B5*SUM(C41:C5040)”

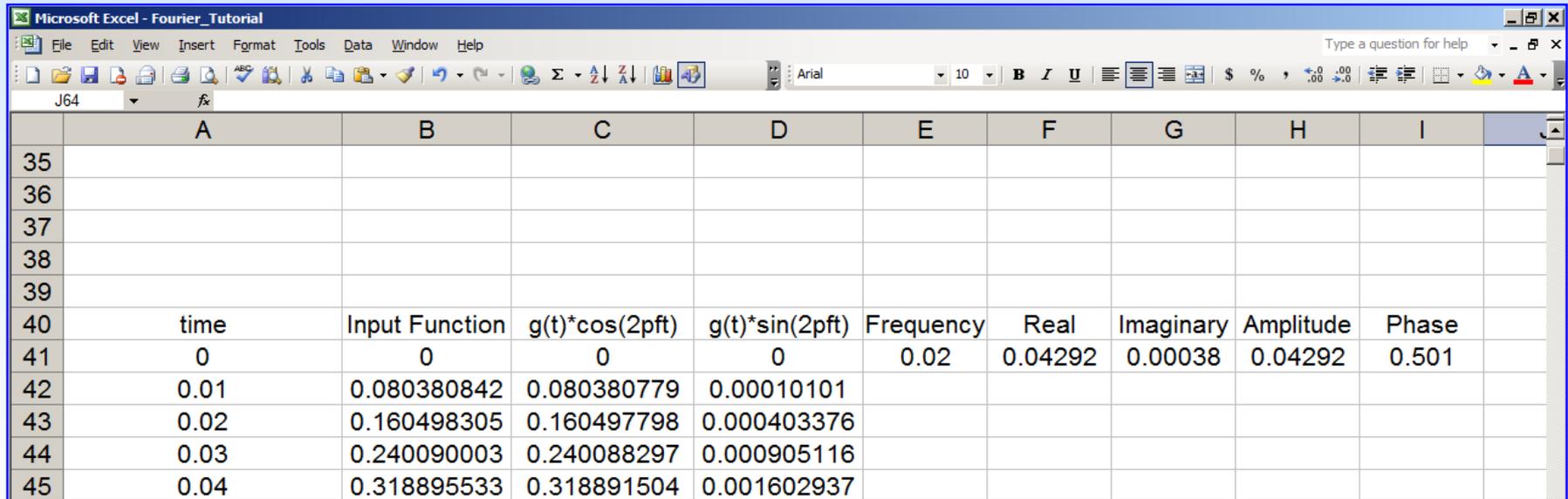
$\text{Im}[G(f_{start} + m \cdot \Delta f)]$ this value will be calculated in cell G41: “=-B5*SUM(D41:D5040)”

Let's calculate the amplitude and the phase of the Fourier transform:

$|G(f)| \approx \sqrt{\text{Re}[G(f)]^2 + \text{Im}[G(f)]^2}$ this value will be calculated in cell H41: “=SQRT(F41^2+G41^2)”

$\text{Phase}[G(f)] = \frac{180}{\pi} \cdot \text{atan}\left(\frac{\text{Im}[G(f)]}{\text{Re}[G(f)]}\right)$ value calculated degrees in cell I41: “=180*ATAN(G41/F41)/PI()”

Overview of the calculation area:



	A	B	C	D	E	F	G	H	I
35									
36									
37									
38									
39									
40	time	Input Function	$g(t) \cdot \cos(2pft)$	$g(t) \cdot \sin(2pft)$	Frequency	Real	Imaginary	Amplitude	Phase
41	0	0	0	0	0.02	0.04292	0.00038	0.04292	0.501
42	0.01	0.080380842	0.080380779	0.00010101					
43	0.02	0.160498305	0.160497798	0.000403376					
44	0.03	0.240090003	0.240088297	0.000905116					
45	0.04	0.318895533	0.318891504	0.001602937					

- Above, there is a snapshot of the spreadsheet calculation area discussed so far.

- Range E41:I41 contains the final Fourier formulas. Everything below (range E42:I5040) will be filled with information from range E41:I41 by a copy-paste type of macro. A reset macro will clear range E42:I5040.

The “Reset()” and “Calculate ()” macros:

- The “Reset()” macro clears all the information from any previous calculation which is in range E42:I5040. The macro also resets the index number.
- The worksheet computes the Fourier transform for a single frequency point. If we were to have the worksheet calculate the transform in few hundred frequency points that would have resulted into an excessively large file.
- A good compromise is to have the “Calculate_()” macro change the frequency from start to end in small increments and save the results in a table of constants. This way we have just a few cell with formulas and the file is kept at a reasonable size. The only problem with this method is that we trade file size for speed.
- Boolean variable “s” has the role of allowing the same button to be used for starting and pausing the macro.
- The “Calculate_()” macro has a loop which achieve two purposes. The first is to take the active calculation results and store them as constants in a table (it does this by a copy-paste operation). The second purpose is to increment the index thus allowing the worksheet to calculate a Fourier function for a new updated frequency point during every loop cycle.
- The “Do” loop is a conditional loop and it can run only if s = True and the index is smaller than the maximum number of points specified in the input parameter area. This achieves two goals, one is that the macro can be started and paused from the same button, and the other is that the macro will run the calculation through the number of frequency points specified and then stop. A new run is then possible only after a reset.

```
Dim s As Boolean
```

```
-----  
Sub Reset()  
[B4] = 0  
[E42:I5040].Clear  
End Sub
```

```
-----  
Sub Calculate_()  
s = Not (s)  
Do While s = True And [B4] < [B3]  
DoEvents  
[B4] = [B4] + 1  
DoEvents  
[E42:I2040] = [E41:I2039].Value  
Loop  
End Sub
```

Charting the Fourier transform components:

- Chart the **input** function of **time** on a 2D scatter chart:

X-Values Range: A41:A5040, Y-Values Range: B41:B5040

- Chart the **amplitude** part of the Fourier transform function of **frequency** on a 2D scatter chart:

X-Values Range: E41:E5040, Y-Values Range: H41:H5040

- Chart the **phase** part of the Fourier transform function of **frequency** on a 2D scatter chart:

X-Values Range: E41:E5040, Y-Values Range: I41:I5040

- Chart the **real** part of the Fourier transform function of **frequency** on a 2D scatter chart:

X-Values Range: E41:E5040, Y-Values Range: F41:F5040

- Chart the **imaginary** part of the Fourier transform function of **frequency** on a 2D scatter chart:

X-Values Range: E41:E5040, Y-Values Range: G41:G5040

