Introduction to Geometrical Optics - a 2D ray tracing Excel model for spherical mirrors - Part 1

- This is a tutorial explaining the creation of an exact 2D ray tracing model for both spherical concave and spherical convex mirrors.

- The model is 2D in the sense that the ray tracing is done in the median x-y plane of symmetry of the mirror

- This is an exact model in the sense that no geometrical approximations are used, however the model does not take into consideration diffraction effects.

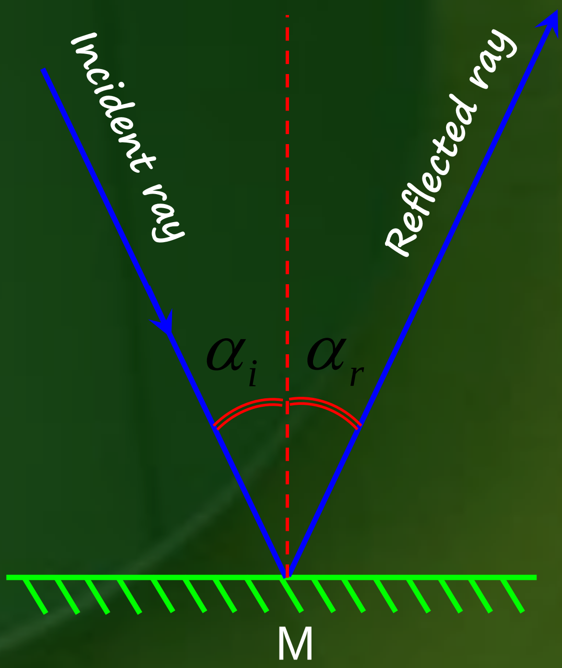
The ideal reflection laws:

There are two reflection laws:

1. The incident ray the normal to the surface and the reflected ray are situated in the same plane

2. The angle of incidence and the angle of reflectance are equal

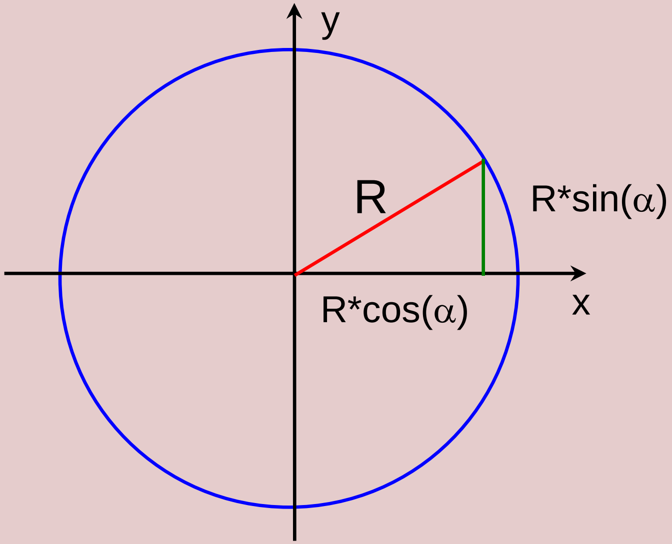
$$\boxed{\alpha_i = \alpha_r}$$



by George Lungu

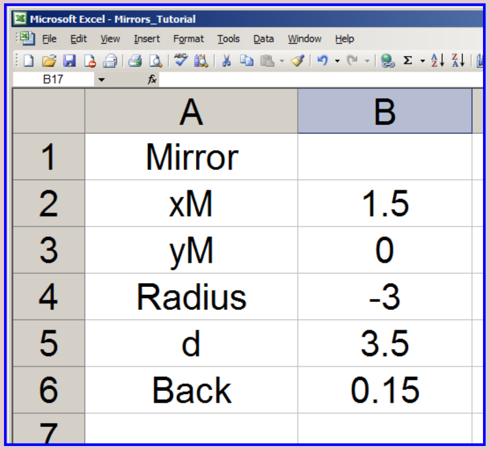
Creating the chart of a spherical mirror section:

- This being a 2D ray tracing program, the spherical mirror will be represented by a section through its x-y median plane which is a circular arc.
- We use a parametric Cartesian representation of the circular arc based on the definition of the trigonometric functions on the trigonometric circle.
- $x = R*\cos(\alpha)$ and $y = R*\sin(\alpha)$, the angle α being proportional to a parameter index.



Conventions and Excel implementation:

- In our model the light will travel from left to right before the reflection.
- By an ad-hoc convention the radius of the curvature will be positive for a convex mirror and negative for a concave mirror
- Column A will contain labels.
- Parameters x_M and y_M are the position of the vertex of the mirror and are located in range B2:B3. We name cell B2 "xM" and B3 "yM".
- The radius is placed in cell B4 and we name that cell "Radius"
- Cell B5 contains the diameter of the mirror and we name that cell "d"
- Cell B6 contains the length of the hachured area behind the reflective surface and we name the cell "Back"



	A	B
1	Mirror	
2	xM	1.5
3	yM	0
4	Radius	-3
5	d	3.5
6	Back	0.15
7		

Creating the spherical mirror:

- Range A42:A62 will contain the index parameter which will have the function of scanning a mirror angle (measured from the center of curvature) starting with the top (d/2) of the mirror and ending with the bottom (-d/2) in 21 steps.

$$x(i) = Radius \cdot \left[1 - \cos\left(\frac{i}{10} \cdot \operatorname{asin}\frac{R}{2d}\right)\right] + x_M$$

$$y(i) = Radius \cdot \sin\left(\frac{i}{10} \cdot \operatorname{asin}\frac{R}{2d}\right) + y_M$$

- While increment "i" varies from -10 to 10 in increments of 1 the (x,y) coordinates described in the formulas above will trace an arc of circle with a size (length) of "d", the radius of "Radius", and the vertex placed at coordinate (x_M,y_M)
- A42: "= –10", A43: "=A42+1" then copy A42 down to A62
- B42: "=Radius*(1-COS((A42/10)*ASIN(d/(2*Radius)))) +xM" then copy B42 down to cell B62
- C42: "=Radius*SIN((A42/10)*ASIN(d/(2*Radius)))+yM" then copy C42 down to cell C62

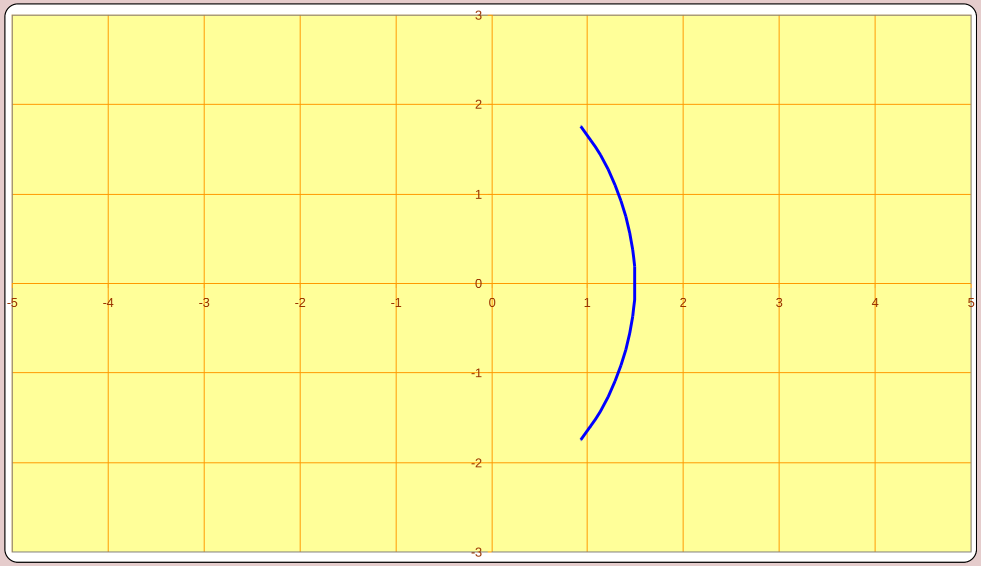
Microsoft Excel - Mirrors

K48

	A	B	C
38			
39			
40		Mirror	
41	Index	x	y
42	-10	0.9367	-1.75
43	-9	1.0409	-1.59494
44	-8	1.13524	-1.4337
45	-7	1.21937	-1.26689
46	-6	1.29295	-1.09518
47	-5	1.3557	-0.91921
48	-4	1.40738	-0.73968
49	-3	1.44778	-0.55729
50	-2	1.47676	-0.37273
51	-1	1.49418	-0.18673
52	0	1.5	0
53	1	1.49418	0.18673
54	2	1.47676	0.37273
55	3	1.44778	0.55729
56	4	1.40738	0.73968
57	5	1.3557	0.91921
58	6	1.29295	1.09518
59	7	1.21937	1.26689
60	8	1.13524	1.4337
61	9	1.0409	1.59494
62	10	0.9367	1.75
63			

Flat | Spherical | Sheet2 | Sheet3

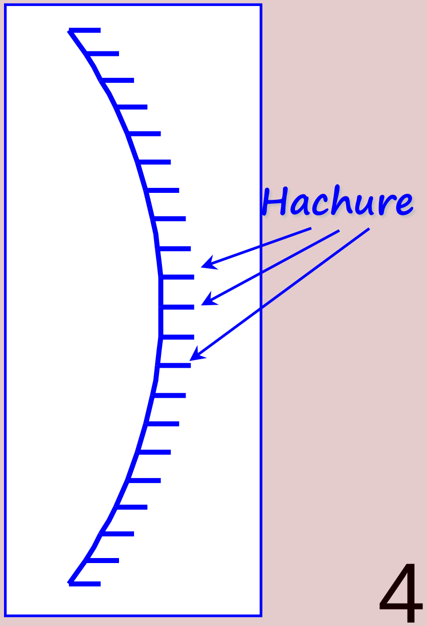
Chart the mirror:



- Select rage B42:C62 => Insert => Chart => Scatter Chart => Finish => delete the legend
- Right click the horizontal axis => Format Axis => Scale => Minimum=-5, Maximum=5, Max Unit=1, Min Unit=1
- Right click the vertical axis => Format Axis => Scale => Minimum=-3, Maximum=3, Max Unit=1, Min Unit=1
- Make the gridlines visible and change their color and the background color to something you like.

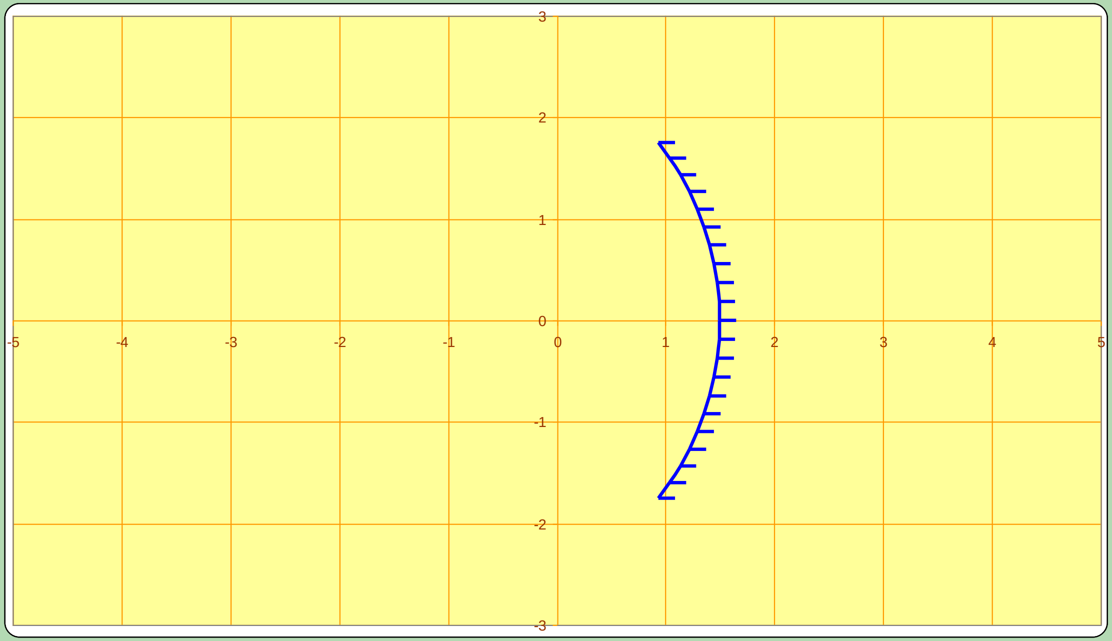
-Double click the curve and make choose the thickest line with no markers. Choose the "Smooth Line" and make sure to stretch the chart so that the grid appears square not rectangular.

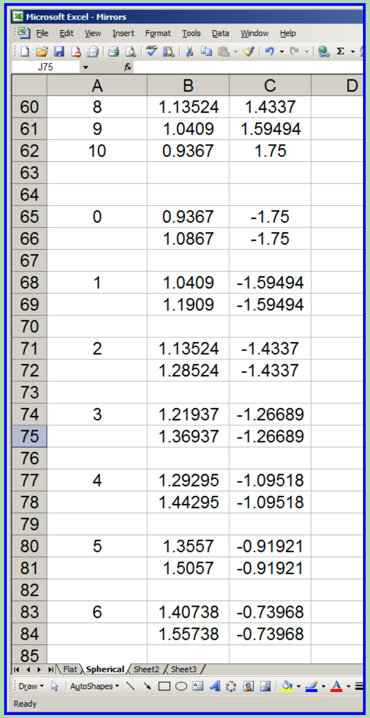
Create the hachure on the back of the mirror:



- We will use the existing coordinates to create the hachure pattern on the back of the mirror.
- A65: "=0", A68: "=A65+1"
- B65: "=OFFSET(B$42,$A65,0)", C65: "=OFFSET(C$42,$A65,0)"
- B66: "=B65+Back", C66: "=C65"
- Copy range B65:C66 into range B68:C69
- Copy range A67:C69 into range A70:C126 and we finished

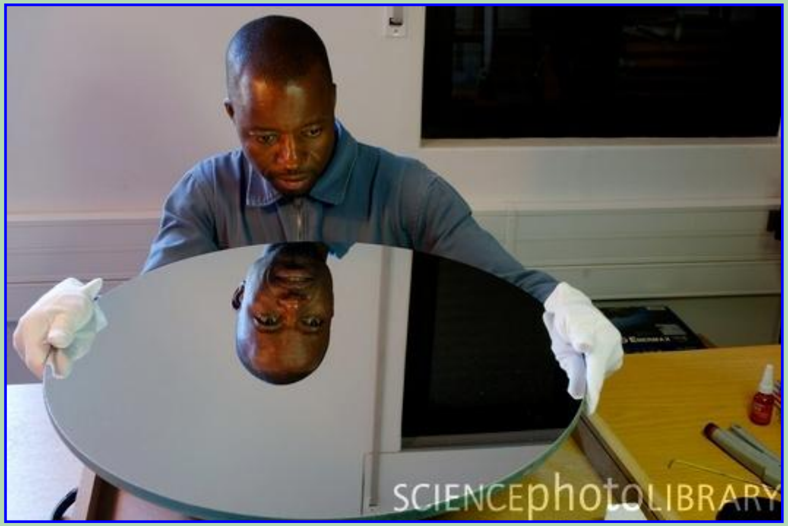
The mirror back – continuation:





	A	B	C	D
60	8	1.13524	1.4337	
61	9	1.0409	1.59494	
62	10	0.9367	1.75	
63				
64				
65	0	0.9367	-1.75	
66		1.0867	-1.75	
67				
68	1	1.0409	-1.59494	
69		1.1909	-1.59494	
70				
71	2	1.13524	-1.4337	
72		1.28524	-1.4337	
73				
74	3	1.21937	-1.26689	
75		1.36937	-1.26689	
76				
77	4	1.29295	-1.09518	
78		1.44295	-1.09518	
79				
80	5	1.3557	-0.91921	
81		1.5057	-0.91921	
82				
83	6	1.40738	-0.73968	
84		1.55738	-0.73968	
85				

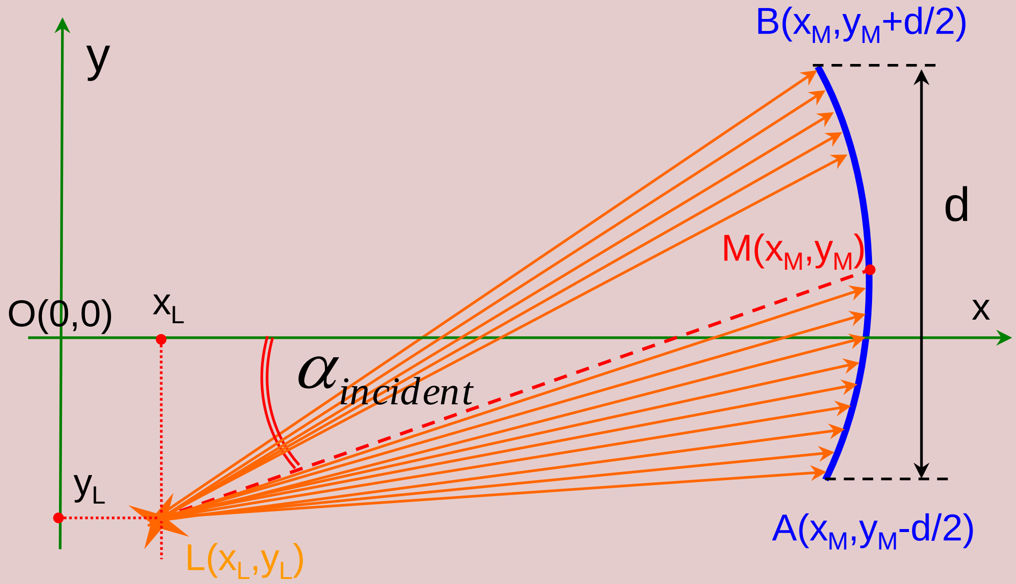
- Extend the data range of the chart from B42:C62 to B42:C1216 and name the series "Reflector".
- Above there is a snapshot of the resulting chart and the hachure formula table.
- Conical mirrors (parabolic, elliptic and hyperbolic) are very close in shape to spherical mirrors and are all used in the construction of astronomical telescopes.



Technician examining a mirror that will be used in the HESS (High Energy Stereoscopic System) array in Namibia. The HESS array is used to investigate gamma ray sources such as supernova remnants and pulsars.

The setup: a brief review of the light source parameters:

- There are many ways of simulating an optical system.
- One of the simplest, yet effective analysis options would be to have a point (artificial star) emitting a bundle of rays towards the mirror and visualizing the bundle behavior after the reflection from the mirror.
- We are interested in both on axis and off axis behavior
- We are also interested in both near and far effects (the star can be within a few focal lengths from the mirror but also thousands of focal lengths from it)
- We would like to keep the angle of incidence constant while we change the x coordinate of the light source.



- Because of this, we will set two input parameters for the light source: x_L and $\alpha_{incident}$.
- We choose to have 21 rays emitted by the star and the rays will be uniformly covering the mirror (there is a constant angle difference between two consecutive rays). The first and the 21st ray hit the edges of the mirror. The diagram above shows only 15 rays out a total of 21.

to be continued...