Longitudinal Aircraft Dynamics #7 – worksheet implementation of the real dynamics

by George Lungu

- This section continues with the dynamics formulas governing our 2D plane and their worksheet implementation.

Some Reynolds number corrections:

- We introduced one single named cell for the Reynolds number (Re) when in fact there are two different Reynolds numbers for the airplane, one for the main wing and one for the horizontal stabilizer. We need two numbers in our model since the wing and the stabilizer have different size chords.
- We have to upgrade the worksheet to account for different Reynolds numbers for the two components.
- First we need to delete the name of the cell B3 in worksheet "Longitudinal_Stability_Model_4".
- Click inside any cell => Insert => Name => Define => find the name you want to delete ("Re" in this case).
- Rename cell B3 "Re_w" => select the cell => => click in the name box in the upper left corner of the worksheet => Type "Re_w" => Hit return.
- Rename cell B5 "Re_s" => select the cell => => click in the name box in the upper left corner of the worksheet => Type "Re_s" => Hit return.
- We also need to update the formulas which included the old "Re" named cell. These formulas are in the range T51:Y61 (see the next page).

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- In cell T51 replace “=IF(Re<$M51,N51,0)” with “=IF(Re_w<$M51,N51,0)”.
- In cell T52 replace “=IF(AND(Re<$M52, Re>=$M51),N51+(N52-N51)*(Re-$M51)/($M52-$M51),0)” with “=IF(AND(Re_w<$M52, Re_w>=$M51),N51+(N52-N51)*(Re_w-$M51)/($M52-$M51),0)” then copy T62 down to T60
- In cell T61 replace “=IF(Re>$M60,N60,0)” with “=IF(Re_w>$M60,N60,0)”
- Copy range T51:T61 to the right up to range V51:V61
- In cell W51 replace “=IF(Re<$M51,Q51,0)” with “=IF(Re_s<$M51,Q51,0)”.
- In cell W52 replace “=IF(AND(Re<$M52, Re>=$M51),Q51+(Q52-Q51)*(Re-$M51)/($M52-$M51),0)” with “=IF(AND(Re_s<$M52, Re_s>=$M51),Q51+(Q52-Q51)*(Re_s-$M51)/($M52-$M51),0)” then copy W62 down to W60
- In cell W61 replace “=IF(Re>$M60,Q60,0)” with “=IF(Re_s>$M60,Q60,0)”
- Copy range W51:W61 to the right up to range Y51:Y61 and you are finished.

The formulas for the Reynolds numbers:

- Reynolds numbers are used to characterize viscosity effects in aerodynamics. As the formula below shows, the number is proportional to the product between the relative speed of the air [m/s] with respect to the body, the length of the body in [m] (airfoil chord in this case) and inversely proportional to the air viscosity which below 4000 meter is roughly constant.

$$\mathrm{Re} = \frac{Speed \cdot Chord}{Viscosity}$$

- The formula can also be roughly expressed as:

$$\mathrm{Re} = Speed \cdot Chord \cdot 70000$$

where the speed is measured in [meter/second] and the length is in meters

The formulas for the Reynolds numbers – worksheet implementation:

- Just like it was mentioned in the previous tutorial we must by any means avoid circular referencing.
- Since the Reynolds numbers are used in the "present" formula area to calculate the aerodynamic forces, momentums, speeds and coordinates the Reynolds values will have to be calculated from "past" speeds (to avoid circular referencing).
- We can use the following formula since we only have available the x and y components of the aircraft speed in range D52:E52:

$$\mathrm{Re}\_w = 70000 \cdot Chord\_wing \cdot v_{plane} = 70000 \cdot Chord\_wing \cdot \sqrt{v_{plane\_x}^2 + v_{plane\_y}^2}$$

- Again, the x and y speed components in the above formulas are and have to be "past" components.
- Similarly we can write the formula for the horizontal stabilizer Reynolds number:

$$\mathrm{Re}\_s = 70000 \cdot Chord\_stabilizer \cdot v_{plane} = 70000 \cdot Chord\_stabilizer \cdot \sqrt{v_{plane\_x}^2 + v_{plane\_y}^2}$$

- B3: "=70000*Chord_wing*SQRT(D52^2+E52^2)"
- B5: "=70000*Chord_stabilizer*SQRT(D52^2+E52^2)"

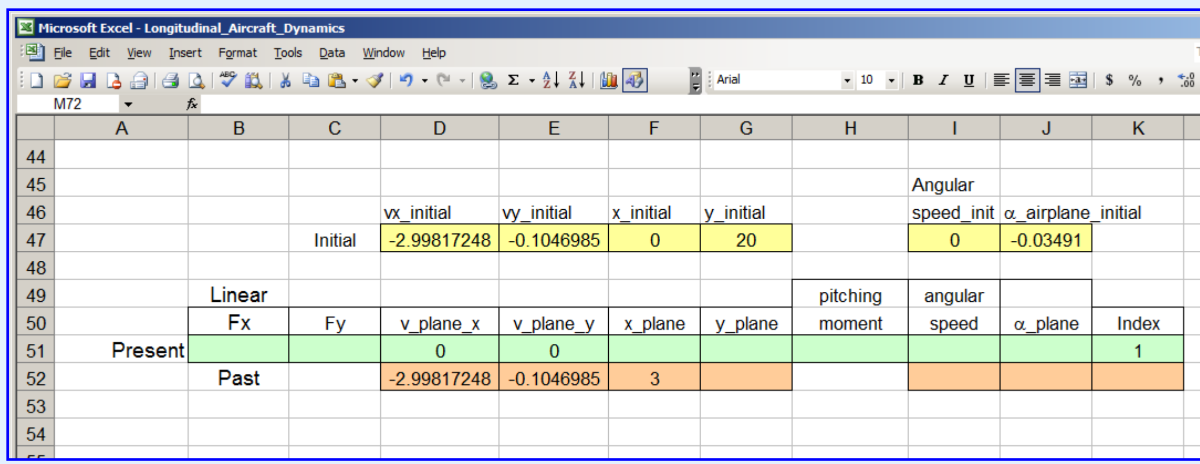
The initial conditions:

- We need to insert the initial linear conditions (initial linear coordinates and linear speed x-y components) formulas in range D47:H47 and the angular initial conditions in range I47:J47 (initial angular speed of the airplane and initial airplane angle).
- We know the initial height, initial magnitude and angle of the initial speed as parameters and we will assume that the initial angle of the airplane is the same as the initial speed angle.

- We have (as input parameters) the initial speed in polar representation (magnitude and angle) but we need to calculate the Cartesian vector components of the initial speed so we need to do a conversion:
- D47: "=-Initial_speed*COS(RADIANS(Initial_angle))" – the minus sign comes from the fact that the airplane is launched from right to left. The argument in the parentheses was being converted to radians since the trigonometric functions treat their arguments as radians only.
- E47: "=Initial_speed*SIN(RADIANS(Initial_angle))"
- F47: "=0" – we will later set up the chart so that the x-scale will span only within negative coordinates and the point of zero x-coordinate will be at the right most of the chart.
- G47: "=Initial_height" – that's an input parameter
- I47: "=0" – we assume the airplane has a null angular (rotation) speed
- J47: "=Initial_angle"
- Please notice that the nature of cell H51 was modified together with the label above it to contain the resultant pitching moment and not the angular acceleration as it did before.

The index calculation:

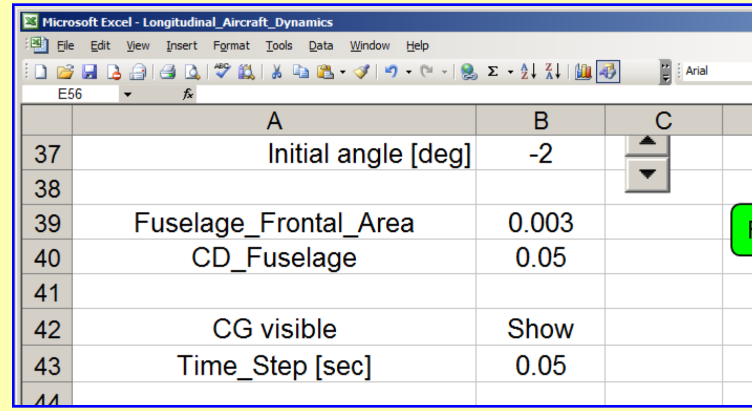
- The index is the integer number counting the numbers of the "Do" loop iterations within the "Run_Pause" macro.
- K51: "=K52+1" – during every loop iteration a copy from K50 to K51 occurs and as a consequence the index value is incremented by 1.


Microsoft Excel - Longitudinal_Aircraft_Dynamics

	A	B	C	D	E	F	G	H	I	J	K
44											
45									Angular		
46				vx_initial	vy_initial	x_initial	y_initial		speed_init	α_airplane_initial	
47			Initial	-2.99817248	-0.1046985	0	20		0	-0.03491	
48											
49		Linear						pitching	angular		
50		Fx	Fy	v_plane_x	v_plane_y	x_plane	y_plane	moment	speed	α_plane	Index
51	Present			0	0						1
52		Past		-2.99817248	-0.1046985	3					
53											
54											

Creating two input entries for the fuselage drag:

- We have not introduced any aerodynamics effects (drag) from the fuselage. We will insert two input parameter entries (in cells B39 and B40), the fuselage frontal area and the aerodynamic coefficient cd (we could adjust both of them).
- I had to move the "Time_step" cell two rows down to make room for the new entries.



Microsoft Excel - Longitudinal_Aircraft_Dynamics

	A	B	C
37	Initial angle [deg]	-2	
38			
39	Fuselage_Frontal_Area	0.003	
40	CD_Fuselage	0.05	
41			
42	CG visible	Show	
43	Time_Step [sec]	0.05	

The formulas for the force calculations:

-An inventory of the forces acting on the aircraft is given below:

=> There are three drag forces always aligned with the speed of the aircraft but of opposite sense.

=> There are two lift forces always perpendicular to the speed of the aircraft.

=> The gravity which is applied in the center of mass and it is always vertical, pointing down.

- Let's write the x-y components of these forces:

Based on these general equations from the previous section:

$$Lift = \frac{1}{2}\cdot\left(c_l \cdot Air\_density \cdot Speed^2 \cdot Chord \cdot Span\right)$$

$$Drag = \frac{1}{2}\cdot\left(c_d \cdot Air\_density \cdot Speed^2 \cdot Chord \cdot Span\right)$$

$$Moment = \frac{1}{2}\cdot\left(c_m \cdot Air\_density \cdot Speed^2 \cdot Chord^2 \cdot Span\right)$$

We can write the wing lift force (below):

$$Lift_{wing} = \frac{c_{l\_wing} \cdot Air\_density \cdot Speed^2 \cdot Chord\_wing \cdot Span\_wing}{2}$$

Knowing that the lift is always perpendicular to the speed of the aircraft vector we can decompose the lift in x and y components:

$$\begin{cases} Lift_{wing\_x} = \dfrac{c_{l\_wing} \cdot Air\_density \cdot (v_{plane\_x}^2 + v_{plane\_y}^2) \cdot Chord\_wing \cdot Span\_wing \cdot \sin(alpha\_speed)}{2} \\ Lift_{wing\_y} = \dfrac{c_{l\_wing} \cdot Air\_density \cdot (v_{plane\_x}^2 + v_{plane\_y}^2) \cdot Chord\_wing \cdot Span\_wing \cdot \cos(alpha\_speed)}{2} \end{cases}$$

The formulas for the Reynolds numbers:

- Using the same general equations, applying them to the main wing and the horizontal stabilizer then decomposing them on the x and y axes we can write the final system (pay attention to the signs):

$$Lift_{wing\_x} = c_{l\_wing} \cdot q_d \cdot Chord\_wing \cdot Span\_wing \cdot \sin(alpha\_speed)$$

$$Lift_{wing\_y} = c_{l\_wing} \cdot q_d \cdot Chord\_wing \cdot Span\_wing \cdot \cos(alpha\_speed)$$

$$Lift_{stabilizer\_x} = c_{l\_stabilizer} \cdot q_d \cdot Chord\_stabilizer \cdot Span\_stabilizer \cdot \sin(alpha\_speed)$$

$$Lift_{stabilizer\_y} = c_{l\_stabilizer} \cdot q_d \cdot Chord\_stabilizer \cdot Span\_stabilizer \cdot \cos(alpha\_speed)$$

$$Drag_{wing\_x} = c_{d\_wing} \cdot q_d \cdot Chord\_wing \cdot Span\_wing \cdot \cos(alpha\_speed)$$

$$Drag_{wing\_y} = -c_{d\_wing} \cdot q_d \cdot Chord\_wing \cdot Span\_wing \cdot \sin(alpha\_speed)$$

$$Drag_{stabilizer\_x} = c_{d\_stabilizer} \cdot q_d \cdot Chord\_stabilizer \cdot Span\_stabilizer \cdot \cos(alpha\_speed)$$

$$Drag_{stabilizer\_y} = -c_{d\_stabilizer} \cdot q_d \cdot Chord\_stabilizer \cdot Span\_stabilizer \cdot \sin(alpha\_speed)$$

$$Drag_{fuselage\_x} = c_{d\_fuselage} \cdot q_d \cdot Frontal\_area\_fuselage \cdot \cos(alpha\_speed)$$

$$Drag_{fuselage\_y} = -c_{d\_fuselage} \cdot q_d \cdot Frontal\_area\_fuselage \cdot \sin(alpha\_speed)$$

$$Gravity_x = 0$$

$$Gravity_y = -g \cdot m$$

Where q is the dynamic pressure expressed as:

$$q_d = \frac{1}{2} \cdot Air\_density \cdot Speed\_airplane^2 = \frac{1}{2} \cdot Air\_density \cdot (v_{plane\_x}^2 + v_{plane\_y}^2)$$

To be continued...